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FUNKSIYANING UZLUKSIZLIK MODULI VA UNING ASOSIY XOSSALARI

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Annotatsiya

$[a, b]$ Kesmada chegaralangan $f(x)$ funksiyaning uzluksizlik moduli va uning asosiy xossalari.

$[a, b]$ da chegaralangan $f(x)$ funksiya berilgan bo'lsin. Ushbu

$$\sup_{\substack{|x_1 - x_2| < \delta \\ a \le x_1, x_2 \le b \\ 0 < \delta \le b-a}} |f(x_2) - f(x_2)| \overset{\text{def}}{=} \omega(f, \delta)$$

funksiyaga $f(x)$ funksiyaning modul uzluksizligi deyiladi.

Takidlaymizki, $\lim_{\delta \to 0} \omega(f, \delta) = 0$ shart $f(x)$ funksiyaning uzluksiz bo'lishi uchun zaruriy va yetarli shart bo'ladi.

$[a, b]$ da uzluksiz bo'lgan funksiyalar sinfini $C_{[a,b]}$ deb belgilaymiz. Biz bundan keyin $f(t) \in C_{[a,b]}$ deb qaraymiz.

Uzluksiz funksiyaning modul uzluksizligi ta'rifidan uning quyidagi xossalari kelib chiqadi:

1^0. $\omega(f, 0) = 0$

2^0. $\omega(f, \delta)$ funksiya δ bo'yicha kamayuvchi.

3^0. $\omega(f, \delta)$ yarim additiv, ya'ni

$\omega(f, \delta_1 + \delta_2) \le \omega(f, \delta_1) + \omega(f, \delta_2)$

4^0. $\omega(f, \delta)$, δ bo'yicha $[a, b]$ da uzluksiz funksiya bo'ladi.

1-ta'rif. Agar $\omega(\delta)$ $(0 < \delta \le l_0 = b - a)$ funksiya $1^0 - 4^0$shartlarni qanoatlantirsa, u holda uzluksiz funksiyaning modul uzluksizligi deyiladi.

1-lemma. $\omega(\delta)$ $(a, l_0]$ da $\uparrow$, $\varphi(\delta) \ge 0$, yarim additiv bo'lsa, u holda

$\forall t_1, t_2 \in (a, l_0]$ uchun $|\varphi(t_1) - \varphi(t_2)| \le \varphi C|t_1 - t_2|$ o'rinli.

$\omega(\delta)$-modul uzluksiz bo'lsin. U holda $\omega(f,\delta) = \omega(\delta)$ bo'ladi.

Haqiqatdan ham

$$\omega(\omega,\delta) = \sup_{|\delta_1-\delta_2|<\delta} |\omega(\delta_1) - \omega(\delta_2)| \le \sup_{|\delta_1-\delta_2|<\delta} \omega(|\delta_1-\delta_2|) \le \omega(\delta)$$

ikkinchi tomondan $\omega(\omega,\delta) \ge \omega(\delta) - \omega(0) = \omega(\delta)$,

ya'ni $\omega(\omega,\delta) = \omega(\delta)$.

2-lemma. Agar $[0, b-a]$ da kamaymovchi, uzluksiz bo'lgan $\omega(\delta)$ funksiya: $(0) = 0$, $\frac{\varphi(\delta)}{\delta}$ o'smovchi bo'lsa, u holda $\omega(\delta)$-modul uzluksiz bo'ldi.

Isbot. $\varphi(\delta)$ funksiya uchun 1^0, 2^0 va 4^0 lar lemmaning shartiga ko'ra bajariladi. 3^0 munosabat quyidagi

$$\varphi(\delta_1+\delta_2) = \delta_1 \cdot \frac{\varphi(\delta_1+\delta_2)}{\delta_1+\delta_2} + \delta_2 \cdot \frac{\varphi(\delta_1+\delta_2)}{\delta_1+\delta_2} \le$$

$$\le \delta_1 \cdot \frac{\varphi(\delta_1)}{\delta_1} + \delta_2 \cdot \frac{\varphi(\delta_2)}{\delta_2} = \varphi(\delta_1) + \varphi(\delta_2).$$

Takidlaymizki, $\frac{\varphi(\delta)}{\delta}$ ning o'smovchi bo'lishlik sharti modul uzluksiz uchun yetarli shart bo'lib hisoblanadi.

Modul uzluksizlikning navbatdagi xossasi:

5^0. Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda $\forall\lambda \in R$ $(\lambda > 0)$ uchun

$$\omega(\lambda\delta) = (\lambda+1)\omega(\delta) \qquad (1)$$

tengsizlik o'rinli bo'ladi.

Isbot. a) $\forall n \in N$ bo'lsin. Bu xossaning isboti 3^0 xossadan bevosita kelib chiqadi, ya'ni

$$\omega(\delta_1+\delta_2) \le \omega(\delta_1) + \omega(\delta_2) \Longrightarrow \delta_1 = \delta_2 = \delta$$

deb olsak,

$$\omega(2\delta) \le 2\,\omega(\delta)$$

Matematik induksiya usuli yordamida $\omega(n\delta) \le n\,\omega(\delta)$ tengsizlikning o'rinli ekanligini ko'rsatish mumkin.

b) $\forall\lambda \in R$ bo'lib, $(\lambda > 0)$ $n < \lambda < n+1$ bo'lsin. Ma'lumki, $\omega(\delta)$-monoton o'suvchi funksiya. U holda $\omega(\lambda\delta) \le \omega(n+1)\delta \le (\lambda+1)\omega(\delta)$.

6^0. $\delta_1 \le \delta_2$ bo'lsin. U holda

$$\omega(\delta_2) = \omega\left(\delta_1 \cdot \frac{\delta_2}{\delta_1}\right) \le \left(\frac{\delta_2}{\delta_1}+1\right)\omega(\delta_1) = \frac{\delta_2}{\delta_1}\left(1+\frac{\delta_1}{\delta_2}\right)\omega(\delta_1) \le 2\frac{\delta_2}{\delta_1}\omega(\delta_1).$$

Keyingi tengsizlikdan $\forall\delta_1 \le \delta_2$ bo'lganda

$$\frac{\omega(\delta_2)}{\delta_2} \le 2\frac{\omega(\delta_1)}{\delta_1} \qquad (*)$$

bo'ladi. Bu xossadan $\frac{\omega(\delta)}{\delta}$ –deyarli kamaymovchi funksiya ekanligi kelib chiqadi.

$\varphi(\delta)$ $(0,l_0]$ da aniqlangan musbat funksiya bo'lsin.

2-ta'rif. Agar $\exists A > 0$ $(A_1 > 0)$ topilib, $\forall 0 < \delta_1 < \delta_2$ lar uchun

$\varphi(\delta_1) \leq A\varphi(\delta_2)$ $(\varphi(\delta_1) \geq A\varphi(\delta_2))$ bo'lsa, u holda $\varphi(\delta)$ funksiya $(0,l_0]$ da deyarli o'suvchi (deyarli kamayuvchi) deyiladi.

3-lemma. Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda

$$\frac{1}{2}\varphi(\delta) \leq \frac{1}{\delta}\int_0^\delta \omega(t)dt \leq \varphi(\delta) \qquad (2)$$

tengsizlik o'rinli bo'ladi.

Isbot. $\omega(\delta)$-modul uzluksiz bo'lsin.$\Longrightarrow \frac{1}{\delta}\int_0^\delta \omega(t)dt \leq \frac{1}{\delta}\int_0^\delta \omega(\delta)dt \leq \omega(\delta)$.

$$\omega(\delta) - \frac{1}{\delta}\int_0^\delta \omega(t)dt \leq \frac{1}{\delta}\int_0^\delta (\omega(\delta) - \omega(t))dt \leq \frac{1}{\delta}\int_0^\delta \omega(\tau)d\tau, \quad [\delta - t = \tau]$$

$$\frac{1}{2}\varphi(\delta) \leq \frac{1}{\delta}\int_0^\delta \omega(\tau)d\tau \ .$$

4-lemma. Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda

$$\frac{1}{x+y}\int_0^{x+y} \omega(t)dt \leq \frac{1}{x}\int_0^x \omega(t)dt + \frac{1}{y}\int_0^y \omega(t)dt \qquad (3)$$

(bunda $0 \leq x, y, x + y \leq l_0$) tengsizlik o'rinli bo'ladi.

Isbot. (3) da $y = \alpha x$ deb olish bilan topamiz:

$$\frac{\int_0^{(1+\alpha)x} \omega(t)\,dt}{1+\alpha} \leq \frac{\int_0^{\alpha x} \omega(t)\,dt}{\alpha} + \int_0^x \omega(t)dt \qquad (4)$$

(4) ning to'g'riligi ushbu

$$\psi(x) = \int_0^x \omega(t)dt + \frac{1}{\alpha}\int_0^{\alpha x} \omega(t)dt - \frac{1}{1+\alpha}\int_0^{(1+\alpha)x} \omega(t)dt$$

funksiyaning $x = 0$ da $\psi(0) = 0$ bo'lishi va uning kamaymovchi ekanligidan kelib chiqadi.

1-teorema. Agar $\omega(\delta)$-modul uzluksiz bo'lsa, u holda

$$\omega^*(\delta) = \frac{1}{\delta}\int_0^\delta \omega(t)dt$$

funksiya ham modul uzluksiz bo'ladi.

Isbot. Agar (2) tengsizlikni e'tiborga olsak, u holda

$$\lim_{\delta \to 0} \omega^*(\delta) = 0$$

ekanligiga ishonch hosil qilish qiyin emas. 2.1.2-lemmaga asosan $(\omega^*(\delta))' \geq 0$, ya'ni $\omega^*(\delta)$-

kamaymovchi funksiya. Ravshnki $\omega^*(\delta)$ –uzluksiz. $\omega^*(\delta)$ ning yarm additivligi 2.1.3-lmmadan kelib chiqadi. Shu bilan teorema isbot bo'ldi.

$\varphi(\delta)$ va $\psi(\delta)$ funksiyalar $(0,l_0]$ da aniqlangan noldan farqli, musbat uzluksiz bo'lsin.

3-ta'rif. Agar $\exists A_1, A_2 > 0$ sonlar mavjud bo'lib, $\forall \delta_1, \delta_2 \in (0,l_0]$ lar uchun

$A_1\psi(\delta) \leq \varphi(\delta) \leq A_2\psi(\delta)$

tengsizlik bajarilsa, u holda $\varphi(\delta)$ va $\psi(\delta)$ funksiyalar $(0,l_0]$ da ekvivalent $(\varphi \sim \psi)$ deyiladi.

4-ta'rif. Agar $\exists A > 0$ soni mavjud bo'lib, $\forall 0 < \delta_1 < \delta_2 < l_0$ lar uchun

$\varphi(\delta_1) \leq A\varphi(\delta_2)$

tengsizlik o'rinli bo'lsa, u holda $\varphi(\delta)$ funksiya $(0,l_0]$ da deyarli o'suvchi deyiladi.

5-ta'rif. Agar $\exists A_1 > 0$ son mavjud bo'lib, $\forall 0 < \delta_1 < \delta_2 < l_0$ lar uchun

$\varphi(\delta_1) \geq A_1\varphi(\delta_2)$

tengsizlik bajarilsa, u holda $\varphi(\delta)$ funksiya $(0,l_0]$ da deyarli kamayuvchi deyiladi.

Takidlaymizki, agar $M \geq \varphi(\delta) \geq \alpha > 0 \ \ (0 \leq \delta \leq 1)$ bo'lsa, u holda u deyarli o'suvchi bo'ladi. Bu holda $= \frac{M}{\alpha}$.

Agar $\varphi(\delta)$-modul uzluksiz bo'lsa, u holda (*) tengsizlikdan $\frac{\varphi(\delta)}{\delta}$ ning deyarli kamayuvchi funksiya bo'lishligi kelib chiqadi.

Ravshanki, agar $\varphi(\delta) \sim \psi(\delta)$ bo'lib, $\varphi(\delta)$ ning deyrli o'suvchi (deyarli kamayuvchi) ligidan $\psi(\delta)$ ning deyarli o'suvchi (deyarli kamayuvchi) ligi kelib chiqadi.

5-lemma. $\varphi(\delta)$ funksiya deyarli o'suvchi (deyarli kamayuvchi) bo'lishligi uchun kamaymovchi (o'smovchi) funksiyaga ekvivalent bo'lishi zarur va yetarlidir.

Isbot. Yetarliligi. $\varphi(\delta)$ funksiya biror kamaymovchi $\psi(\delta)$ funksiyaga ekvivalent, ya'ni $\varphi(\delta) \sim \psi(\delta)$ bo'lsin. 2.1.2-ta'rifga ko'ra $\exists A_1, A_2 > 0$ sonlari mavjud bo'lib, $\forall 0 < \delta \leq l_0$ lar uchun

$A_1\psi(\delta) \leq \varphi(\delta) \leq A_2\psi(\delta)$ tengsizlik bajariladi. $\delta_2 < \delta_1$ bo'lsin. U holda yuqoridagi tengsizlikdan

$\varphi(\delta_2) \leq A_2\psi(\delta_2) \leq A_2\psi(\delta_1) \leq A_2\frac{\varphi(\delta_1)}{A_1} = \frac{A_2}{A_1}\varphi(\delta_1)$ bo'ladi. Demak, $\varphi(\delta)$ funksiya deyarli o'suvchi.

Zarurligi. $\varphi(\delta)$ deyarli o'suvchi funksiya bo'lsin, ya'ni $\forall 0 < \delta_1 < \delta_2 \in (0,l_0]$ lar uchun

$$\varphi(\delta_1) \leq A\varphi(\delta_2) \tag{5}$$

ushbu

$$\psi(\delta) = \sup_{0 < \eta < \delta}\varphi(\eta) \tag{6}$$

funksiyani tuzamiz.

Ravshanki, $\psi(\delta)$ funksiya kamaymovchi. Endi kamaymoovchi $\psi(\delta)$ funksiyaning $\varphi(\delta)$ funksiyaga ekvivalentligini ko'rstamiz: $\psi(\delta)$ funksiyaning tuzulishidan $\varphi(\delta) \leq \psi(\delta)$. (5) dan $\forall\eta\ (0 < \eta < \delta)$ uchun

$$\psi(\eta) \leq A\varphi(\delta) \Rightarrow \psi(\delta) = \sup_{0<\eta<\delta} \varphi(\eta) \leq A\,\varphi(\delta), \psi(\delta) \leq A\varphi(\delta).$$

(6) ni e'tiborga olib topamiz: $\frac{1}{A}\psi(\delta) \leq \varphi(\delta) \leq \psi(\delta)$ tengsizlik bajariladi. Keyingi tengsizlikdan $\varphi(\delta) \sim \psi(\delta)$ ekanligi kelib chiqadi.

Xuddi shunday yo'l bilan lemmaning ikkinchi qismi ham isbot qilindi, ya'ni : $\varphi(\delta)$ funksiya deyarli kamayuvchi bo'lishligi uchun uning biror o'smovchi funksiyaga ekvivalent bo'lishi zarur va yetarlidir.

Xulosa.

Ushbu maqola umumlashgan Gyolder fazosida funksiyaning uzluksizlik moduli va uning asosiy xossalari o'rganishga bag'ishlangan.

Foydalanilgan adabiyotlar.

1. В. А. Какичев. «Граничные свойства интеграла типа Коши многих переменных». Уч. зап. Шахт. пед. Ин-та. 2, вып. 6, 1959-г, 25-90 с.
2. Ф. Д. Гахов. «Краевые задачи». Москва. Наука. 1977-г.
3. Н. И. Мусхелишвили. «Сингулярные интегральные уравнения». Москва. Наука. 1968-г.
4. А. Газиев. «Поведение интеграла типа Коши вблизи границы полицилиндрической области». Самарканд-1987.