https://gospodarkainnowacje.pl GOSPODARKA I INNOWACJE
LABORATORIUM WIEDZY Artur Borcuch
Volume: 33 | 2023
Economy and Innovation
ISSN: 2545-0573
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HAQIQIY O'Q BO'YICHA OLINGAN KOSHI TIPIDAGI INTEGRAL VA UNING XOSSALARI

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A R T I C L E I N F O.

Tayanch iboralar: cheksiz uzoq nuqta, cheksiz uzoq nuqta atrofidagi Geyolder sharti, bosh qiymat, Soxotiskiy formulasi, Koshitipidagi integral, cheksiz uzoq nuqta atrofi, Koshiformulasi, yarim tekislik, Koshitipidagi integral limitik qiymati.

Annotatsiya

Ushbu maqolada Koshi tipidagi integral va uning muhim xossalari, Koshi tipidagi integralning bosh qiymati tog'risidagi tushunchalarni hosil qilish hamda Geyolder sinfi to'g'risida amaliy masalalarga qo'llashni talqin qilamiz.

Teorema. Agar φ(t) funksiya yuqori yarim tekislikda analitik, yopiq yarim tekislikda uzluksiz va cheksiz uzoq nuqta atrofida $|\varphi(\tau)-\varphi(\infty)| \le \frac{A}{|\tau|^{\mu}}$, (μ>0, A>0) shartni qanoatlantirsa, u holda

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)}{\tau - z}d\tau = \begin{cases} \varphi(z)-\frac{\varphi(\infty)}{2}, & \operatorname{Im} z > 0 \\ -\frac{\varphi(\infty)}{2}, & \operatorname{Im} z < 0 \end{cases} \quad (1)$$

Isbot.Yuqori yarim tekislikda markazi koordinata boshida radiusi R ga teng bo'lgan yarim aylana C_R va [-R,R] kesmani qaraymiz. R ni shunday katta qilib olmizki, natijada qaralayotgan nuqta C_R yarim doiraning ichida yotsin. Koshi formulasi va Koshi teoremasiga asosan

$$\frac{1}{2\pi i}\int_{[-R,R]+C_R}\frac{\varphi(\tau)-\varphi(\infty)}{\tau - z}d\tau = \frac{1}{2\pi i}\int_{-R}^{R}\frac{\varphi(\tau)-\varphi(\infty)}{\tau - z}d\tau + \frac{1}{2\pi i}\int_{C_R}\frac{\varphi(\tau)-\varphi(\infty)}{\tau - z}d\tau =$$

$$= \begin{cases} \varphi(z)-\varphi(\infty), \operatorname{Im} > 0 \\ 0, \quad \operatorname{Im} < 0 \end{cases} \quad (2)$$

$|\varphi(\tau)-\varphi(\infty)| \le \frac{A}{|\tau|^{\mu}}$, (μ>0, A>0) formulaga asosan R ning o'sishi bilan integral ostidagi funksiya $\frac{1}{R^{1+\mu}}$ tartib bilan nolga intiladi, chiziqning uzunligi esa R bilan proporsional ravishda o'sadi, shuning uchun S_R bo'yicha olingan integral nolga intiladi. $R\to\infty$ da limitga o'tsak

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)-\varphi(\infty)}{\tau-z}d\tau=\begin{cases}\varphi(z)-\varphi(\infty), & \operatorname{Im}>0\\ 0, & \operatorname{Im}<0\end{cases} \quad (3)$$

Bu yerdan $\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)}{\tau-z}d\tau=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)-\varphi(\infty)}{\tau-z}d\tau+\frac{1}{2}\varphi(\infty)$ formulani e'tiborga olsak, unda (1) formulani hosil qilamiz.

Agar φ(t) pastki tekislikda yuqoridagi teoremaning shartlarini qanoatlanitirsa, xuddi shunday

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)-\varphi(\infty)}{\tau-z}d\tau=\begin{cases}\frac{\varphi(\infty)}{2}, & \operatorname{Im}z>0\\ -\varphi(z)+\frac{\varphi(\infty)}{2}, & \operatorname{Im}z<0\end{cases} \quad (4)$$

formulani isbot qilish mumkin.

(1) va (4) formulalarning o'rinli bo'lishi uchun φ(t) funksiya istalgancha katta t uchun $|\varphi(\tau)-\varphi(\infty)|\le\frac{A}{|\tau|^{\mu}}$, (μ>0, A>0) shartning cheksiz uzoq nuqtaning butun atrofida bajarilishi shart bo'lmasdan, haqiqiy o'qda bajarilishi yetarligini ko'rsatish mumkin.

Bo'lakli analitik funksiyaning F(z)=$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)}{\tau-z}d\tau$ shaklidagi Koshi tipidagi integral bilan ifodalanishi uchun $F^+(\infty)+F^-(\infty)=0$ shartning bajarilishi zarur edi. Bu shartning yetarli ekanini kursatamiz. Faraz qilaylik $|\varphi(\tau)-\varphi(\infty)|\le\frac{A}{|\tau|^{\mu}}$, (μ>0, A>0) va $F^+(t)=(1-\frac{\alpha}{2\pi})\varphi(t)+\frac{1}{2\pi i}\int\frac{\varphi(\tau)}{\tau-t}d\tau$ shartlar o'rinli bo'lsin. Unda $\varphi(\tau)=F^+(\tau)-F^-(\tau)$ desak $\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)}{\tau-z}d\tau=\begin{cases}\varphi(z)-\frac{\varphi(\infty)}{2}, & \operatorname{Im}z>0\\ -\frac{\varphi(\infty)}{2}, & \operatorname{Im}z<0\end{cases}$ va $F^+(\infty)+F^-(\infty)=0$ dan

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)}{\tau-z}d\tau=\begin{cases}\Phi^+(z), & \operatorname{Im}z>0\\ \Phi^-(z), & \operatorname{Im}z<0\end{cases},$$

ya'ni F(z)= $\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\varphi(\tau)}{\tau-z}d\tau$

Xuddi shunday tekshirishni cheksizlikka ketuvchi silliq chiziq bo'yicha olingan Koshi tipilagi integral uchun ham olib borish mumkin.

Teorema.

Agar γ- silliq chiziq bo'lib φ(τ)∈H(λ) bo'lsa, u holda Koshi tipidagi integralning limitik qiymatlari $F^+(t),F^-(t)$ lar ham λ ko'rsatkich bilan Gelder shartini qanoatlantiradi, agarda λ<1 bo'lsa, λ=1 bo'lganda esa Gelder kursatkichi λ-dan istalgancha kichik songa farq qiladi.

Isbot.

$\int_\gamma \frac{\varphi(\tau)}{\tau - t} d\tau = \int_\gamma \frac{\varphi(\tau) - \varphi(t)}{\tau - t} d\tau + \pi i \varphi(t)$ formulaga asosan teoremani isbot qilish uchun $\psi(t) = \frac{1}{2\pi i} \int_\gamma \frac{\varphi(\tau) - \varphi(t)}{\tau - z} d\tau$ funksiyaning Gelder shartini qanoatlantirishini ko'rsatish yetarli $\forall t_1, t_2 \in \gamma$ nuqtalarni olib

$$\psi(t_2) - \psi(t_1) = \frac{1}{2\pi i} \int_\gamma \left[\frac{\varphi(\tau) - \varphi(t_2)}{\tau - t_2} - \frac{\varphi(\tau) - \varphi(t_1)}{\tau - t_1}\right] d\tau \quad (5)$$

ayirmani baholaymiz. t_1 nuqtani markaz qilib, istalgancha kichik δ radiusli aylana chizaylikki, natijada u γ-chiziqni ikki nuqtada kesib o'tsin. γ-chiziq bilan aylananing kesishish nuqtalarini *a* va b bilan belgilaymiz. γ-chiziqning aylana ichidagi kismini γ_δ deb belgilaymiz, kolgan kismini esa γ/γ_δ deb belgilaymiz.Faraz qilaylik t_2 nuqta γ yoydagi a va v nuqtalardan farkli ixtiyoriy belgilangan nuqta bo'lsin.

$\delta = k|t_2 - t_1|$ deb belgilaylik , bunda albatta k>1.γ-dagi t va τ nuqtalar orasidagi yoylarning eng kichigini $s = s(t, \tau)$ deb belgilaymiz.γ-chizik sillik bulgani uchun

$$s(t_1, t_2) \le m|t_2 - t_1| \quad (6)$$

tengsizlik urinli buladi , m-uzgarmas musbat son (5). Ayirmani ushbu kurinishda yozib olamiz:

$$\psi(t_2) - \psi(t_1) =$$

$$= \frac{1}{2\pi i} \int_{\gamma_\delta} \frac{\varphi(\tau) - \varphi(t_2)}{\tau - t_2} d\tau - \frac{1}{2\pi i} \int_{\gamma_\delta} \frac{\varphi(\tau) - \varphi(t_1)}{\tau - t_1} d\tau + \frac{1}{2\pi i} \int_{\gamma/\gamma_\delta} \left[\frac{\varphi(\tau) - \varphi(t_2)}{\tau - t_2} - \frac{\varphi(\tau) - \varphi(t_1)}{\tau - t_1}\right] d\tau =$$

$$= \frac{1}{2\pi i} \int_{\gamma_\delta} \frac{\varphi(\tau) - \varphi(t_2)}{\tau - t_2} d\tau - \frac{1}{2\pi i} \int_{\gamma_\delta} \frac{\varphi(\tau) - \varphi(t_1)}{\tau - t_1} d\tau + \frac{1}{2\pi i} \int_{\gamma/\gamma_\delta} \frac{\varphi(t_1) - \varphi(t_2)}{\tau - \tau_1} d\tau +$$ Bu

$$+ \frac{1}{2\pi i} \int_{\gamma/\gamma_\delta} \frac{[\varphi(\tau) - \varphi(t_2)](t_2 - t_1)}{(\tau - t_2)(\tau - t_1)} d\tau \overset{def}{=} I_1 + I_2 + I_3 + I_4$$

qo'shiluvchilarning xar birini ayrim –ayrim baholaymiz

$$I_2 = \le \frac{1}{2\pi i} \int_{\gamma_\delta} \frac{|\varphi(\tau) - \varphi(t_1)|}{|\tau - t_1|} |d\tau| \le \frac{Am}{2\pi} \int_{\gamma_\delta} |\tau - t_1|^{\lambda - 1} |d\tau| ,$$

γ- chiziqning silliqliligidan (3.18) formula o'rinli, ya'ni $|d\tau| = |ds| \le m|dr|$. Bu tengsizlikni e'tiborga olsak, keyingi tengsizlikdan

$$|I_2| \le \frac{Am}{\pi} \int_0^\delta r^{\lambda - 1} dr = A_1 |t_2 - t_1|^\lambda$$

Xuddi shunday usul bilan I_1 ham baholanadi:

$|I_1| \le A_2 |t_2 - t_1|^{\lambda}$.

Endi I_3 ni baholaymiz:

$$\left|I_3\right| \le \frac{|\varphi(t_1) - \varphi(t_2)|}{2\pi} \left| \int_{\gamma / \gamma_\delta} \frac{d\tau}{\tau - t_1} \right| \le \frac{A|t_2 - t_1|^{\lambda}}{2\pi} \left| \int_{\gamma / \gamma_\delta} \frac{d\tau}{\tau - t_1} \right|$$

$\left| \int_{\gamma / \gamma_\delta} \frac{d\tau}{\tau - t_1} \right| = \ln \frac{a - t_1}{b - t_1}$. $\forall t_1 \in \gamma$ lar uchun $\left| \ln \frac{a - t_1}{b - t_1} \right| \le M$.

Demak, $|I_3| \le \frac{AM}{2\pi} |t_2 - t_1|^{\lambda} \le A_3 |t_2 - t_1|^{\lambda}$.

Eng qiyin bo'lgan I_4 ni baholaymiz. $\varphi(\tau) \in H(\lambda)$ bo'lgani uchun xamda (3.1.8) tengsizlikdan foydalanib, $|I_4| \le \frac{A|t_2 - t_1|}{2\pi} \int_{\gamma / \gamma_\delta} \frac{|d\tau|}{|\tau - t_1||\tau - t_2|^{1-\lambda}} \le \tilde{A}|t_2 - t_1| \int_{\gamma / \gamma_\delta} |\tau - t_1|^{\lambda - 2} \left| \frac{\tau - t_1}{\tau - t_2} \right|^{1-\lambda} |d\tau|$, $\left|\tau - t_1\right| \ge \delta = K|t_2 - t_1|$ bo'lgani uchun $k|\tau - t_2| \ge k[|\tau - t_1| - |t_2 - t_1|] \ge (k-1)|\tau - t_1|$

bo'ladi. Bularni e'tiborga olgan holda $|I_4| \le \tilde{\tilde{A}}(\frac{k}{k-1})^{1-\lambda} |t_1 - t_2| \int_R^{\delta} r^{\lambda - 2} dr$ ga ega bo'lamiz, bunda $R = \max_{\tau \in \gamma / \gamma_\delta} |\tau - t_1|$.

Agar $\lambda < 1$ bo'lsa, u holda keyingi integralni hisoblab $|I_4| \le A_4 |t_2 - t_1|^{\lambda}$

ekanini topamiz.

Agar $\lambda = 1$ bo'lsa, u xolda yana keyingi integralni xisoblash natijasida

$|I_4| \le \tilde{A}_4 |t_2 - t_1|^1 \ln|t_2 - t_1|$ ga ega bo'lamiz. Keyingi tengsizlikdan

$|I_4| \le \tilde{A}_4 |t_2 - t_1|^{1-\varepsilon}$, chunki $|t_2 - t_1|^{\varepsilon} \ln|t_2 - t_1| \le M$

$I_1, I_2, I_3 \text{ва} I_4$ larning bahosini e'tiborga olsak, $\lambda = 1$ bo'lganda esa I_1, I_2, I_3 lardagi λ ni $1-\varepsilon$ ga almashtirish mumkinligini e'tiborga olsak, u vaqtda teoremaning isbot bo'lganligiga ishonch hosil qilamiz.

Bu isbot qilingan teoremadan Koshi integralining quyidagi xossasi kelib chiqadi.

Agar $\varphi(t)$ funksiya γ- yopiq silliq chiziqda λ ko'rsatkich bilan Gelder shartini qanoatlantirsa, u xolda $F(t) = \frac{1}{2\pi} \int_{\gamma} \frac{\varphi(\tau)}{\tau - t} d\tau$ funksiya, agar $\lambda < 1$ bo'lsa, shu ko'rsatkich bilan, agar $\lambda = 1$ bo'lsa, esa $1-\varepsilon$ ko'rsatkich bilan Gelder shartini qanoatlantiradi.

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