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BOSH TO'PLAMNING MATEMATIK KUTILISHINI TURLI HAJMLI TANLANMALAR BO'YICHA BAHOLASH UCHUN ISHONCHLI INTERVALLAR

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ARTICLEINFO.

Kalit so'zlar: Bosh to'plam, dispersiya, normal taqsimot, St'yudent taqsimoti, ishonchli interval.

Annotatsiya

Ushbu ish ilmiy va nazariy xulosalar hosil qilish maqsadida kuzatish natijasida to'plangan ma'lumotlarga asoslanib, taqsimotning noma'lum parametrini baholash usullarini ko'rsatib berishga bog'ishlangan.

Normal taqsimlangan bosh to'plamdan erkli ikkita n_1 va n_2 hajmli tanlanma olingan. $\overline{X_1}$ va $\overline{X_2}$ - bu tanlanmalar bo'yicha tanlanma o'rtacha qiymatlar, S_1^2 va S_2^2 esa tanlanma dispersiyalar bo'lsin. n_1+n_2 lar hajmli birlashtirilgan tanlanma bo'yicha tanlanma o'rtacha qiymat va tanlanma dispersiya

$$\overline{X}=\frac{n_1\overline{X_1}+n_2\overline{X_2}}{n_1+n_2},\ S^2=\frac{(n_1-1)S_1^2+(n_2-1)S_2^2}{n_1+n_2-2}$$

formulalar bo'yicha hisoblanadi.

Bosh to'plamning noma'lum μ matematik kutilishi uchun berilgan ishonchlilik ehtimolida ishonchli intervallar qurish talab etiladi.

Bu masalani yechishda ikki holni qarymiz.

1. Bosh to'plam dispersiyasi σ^2 ma'lum bo'lgan hol. $X_1, X_2, ..., X_{n_1}$ va $Y_1, Y_2, ..., Y_{n_2}$ lar shu bosh to'plamdan olingan n_1 va n_2 hajmli tanlanmalar bo'lsin.

Ushbu $Z=\frac{\overline{X}-\mu}{\sigma}\sqrt{n_1+n_2}$ tasodifiy miqdorlarni qaraymiz. Agar bosh to'plam normal taqsimlangan bo'lsa, u holda erkli kuzatishlar bo'yicha topilgan

$$\overline{X_1}=\frac{1}{n_1}\sum_{i=1}^{n_1}X_i,\ \overline{X_2}=\frac{1}{n_2}\sum_{i=1}^{n_2}X_i,$$

tanlanma o'rtacha qiymatlar ham normal taqsimlangan bo'ladi. $\overline{X_1}$ taqsimotning parametrlari

$M\overline{X_1} = \mu,\ \ \sigma_{x_1} = \sqrt{D\overline{X_1}} = \dfrac{\sigma}{\sqrt{n_1}}$, $\overline{X_2}$ tasqimotning parametrlari esa

$M\overline{X_2} = \mu,\ \ \sigma_{x_2} = \sqrt{D\overline{X_2}} = \dfrac{\sigma}{\sqrt{n_2}}$ bo'ladi.

Haqiqatdan ham $X_i(Y_i)$ qiymatlarning har biri va bosh to'plam belgisi bir xil taqsimotga ega ekanini e'tiborga olsak , bularning va bosh to'plam belgisining son harakteriskalari bir xil bo'ladi. Jumladan $X_i(Y_i)$ miqdorlarning har birining matematik kutilishi bosh to'plam belgisining matematik kutilishiga dispersiyasi esa bosh to'plam belgisining dispersiyasiga, ya'ni

$$MX_i = \mu,\ MY_i = \mu,\ Dx_i = \sigma^2,\ DY_i = \sigma^2.$$

U holda

$$M\overline{X_1} = M\left(\frac{1}{n_1}\sum_{i=1}^{n_1} X_i\right) = \frac{1}{n_1} M\left(\sum_{i=1}^{n_1} X_i\right) =$$

$$= \frac{1}{n_1}\sum_{i=1}^{n_1} MX_i = \frac{1}{n_1}\cdot n_1\mu = \mu,$$

$$\sigma_{x_1}^2 = D\overline{X_1} = D\left(\frac{1}{n_1}\sum_{i=1}^{n_1} X_i\right) = \frac{1}{n_1^2} D\left(\sum_{i=1}^{n_1} X_i\right) =$$

$$= \frac{1}{n_1^2}\sum_{i=1}^{n_1} DX_i = \frac{1}{n_1^2}\cdot n_1\sigma^2 = \frac{\sigma^2}{n_1},\quad \sigma_{x_1} = \frac{\sigma}{\sqrt{n_1}}$$

Huddi shunday

$$M\overline{X_2} = M\left(\frac{1}{n_2}\sum_{i=1}^{n_2} Y_i\right) = \frac{1}{n_2} M\left(\sum_{i=1}^{n_2} Y_i\right) =$$

$$= \frac{1}{n_2}\sum_{i=1}^{n_2} MY_i = \frac{1}{n_2}\cdot n_2\mu = \mu,$$

$$\sigma_{x_2}^2 = D\overline{X_2} = D\left(\frac{1}{n_2}\sum_{i=1}^{n_2} Y_i\right) = \frac{1}{n_2^2} D\left(\sum_{i=1}^{n_2} Y_i\right) =$$

$$= \frac{1}{n_2^2}\sum_{i=1}^{n_2} DY_i = \frac{1}{n_2^2}\cdot n_2\sigma^2 = \frac{\sigma^2}{n_2},\quad \sigma_{x_2} = \frac{\sigma}{\sqrt{n_2}}$$

Shuningdek, $\overline{X}$ tasodifiy miqdor ham normal taqsimlangan bo'lib, $\overline{X}$ taqsimotning parametrlari $M\overline{X} = \mu,\ D\overline{X} = \dfrac{\sigma^2}{n_1 + n_2}$ bo'ladi.

Haqiqatdan

$$M\overline{X} = M\left(\frac{n_1\overline{X_1} + n_2\overline{X_2}}{n_1 + n_2}\right) = \frac{1}{n_1 + n_2} M(n_1\overline{X_1} + n_2\overline{X_2}) =$$

$$= \frac{1}{n_1 + n_2}[M(n_1\overline{X_1}) + M(n_2\overline{X_2})] = \frac{1}{n_1 + n_2}(n_1 M\overline{X_1} + n_2 M\overline{X_2}) =$$

$$= \frac{1}{n_1 + n_2}(n_1\mu + n_2\mu) = \mu,$$

$$D\overline{X} = D\left(\frac{n_1\overline{X_1} + n_2\overline{X_2}}{n_1 + n_2}\right) = \frac{1}{(n_1 + n_2)^2} D(n_1\overline{X_1} + n_2\overline{X_2}) =$$

$$= \frac{1}{(n_1 + n_2)^2}[D(n_1\overline{X_1}) + D(n_2\overline{X_2})] = \frac{1}{(n_1 + n_2)^2}(n_1^2 D\overline{X_1} + n_2^2 D\overline{X_2}) =$$

$$= \frac{1}{(n_1 + n_2)^2}\left(n_1^2 \cdot \frac{\sigma^2}{n_1} + n_2^2 \cdot \frac{\sigma^2}{n_2}\right) = \frac{\sigma^2}{n_1 + n_2}$$

Z tasodifiy miqdor ham $\overline{X}$ normal argumentning chiziqli funksiyasi sifatida normal taqsimotga ega bo'lib, tasqimotning parametrlari $MZ = 0,\ \ DZ = 1$ bo'ladi.

Haqiqatdan

$$MZ = M\left(\frac{\overline{X} - \mu}{\sigma}\sqrt{n_1 + n_2}\right) = \frac{\sqrt{n_1 + n_2}}{\sigma} M(\overline{X} - \mu) =$$

$$= \frac{\sqrt{n_1 + n_2}}{\sigma}(M\overline{X} - \mu) = \frac{\sqrt{n_1 + n_2}}{\sigma}(\mu - \mu) = 0,$$

$$DZ = D\left(\frac{\overline{X} - \mu}{\sigma}\sqrt{n_1 + n_2}\right) = \frac{n_1 + n_2}{\sigma^2} D(\overline{X} - \mu) =$$

$$= \frac{n_1 + n_2}{\sigma^2}(D\overline{X} - D\mu) = \frac{n_1 + n_2}{\sigma^2}(D\overline{X} - 0) =$$

$$= \frac{n_1 + n_2}{\sigma^2} \cdot \frac{\sigma^2}{n_1 + n_2} = 1$$

Undan tashqari Z tasodifiy miqdor μ ning funksiyasi sifatida uzluksiz va manoton hamdir. Ushbu

$$P(U_{\alpha/2} < Z < U_{1-\alpha/2}) = P\left(U_{\alpha/2} < \frac{\overline{X} - \mu}{\sigma}\sqrt{n_1 + n_2} < U_{1-\alpha/2}\right) = 1 - \alpha$$

munosabat bajarilishini talab qilamiz, bu yerda $U_{\alpha/2}$ va $U_{1-\alpha/2}$ - matematik kutilishi 0 ga, dispersiyasi esa 1 ga teng bo'lgan normal taqsimotning kvantillari.

Qavs ichidagi

$$U_{\alpha/2} < \frac{\overline{X} - \mu}{\sigma}\sqrt{n_1 + n_2} < U_{1-\alpha/2}$$

tengsizlikni μ ga nisbatan yechib, $1-\alpha$ ehtimol bilan

$$\overline{X} - \frac{\sigma}{\sqrt{n_1 + n_2}}U_{1-\alpha/2} < \mu < \overline{X} - \frac{\sigma}{\sqrt{n_1 + n_2}}U_{\alpha/2}$$

tengsizlik bajarilishini hosil qilamiz.

Normal taqsimotning kvantillari $U_{\alpha/2} = -U_{1-\alpha/2}$ bog'lanishda bo'lganidan, μ uchun hosil qilingan ishonchli intervalni quyidagicha yozish mumkin:

$$\overline{X} - \frac{\sigma}{\sqrt{n_1 + n_2}}U_{1-\alpha/2} < \mu < \overline{X} + \frac{\sigma}{\sqrt{n_1 + n_2}}U_{1-\alpha/2}.$$

2. Bosh to'plam dispersiyasi σ^2 noma'lum bo'lgan hol. Ravshanki, bu holda oldingi band natijalaridan foydalanib bo'lmaydi, chunki u yerda σ^2 ma'lum deb faraz qilinga edi. Hozirgi holda markaziy statistika sifatida

$$T = \frac{\overline{X} - \mu}{S}\sqrt{n_1 + n_2}$$

tasodifiy miqdor qabul qilinadi. T tasodifiy miqdor $k = n_1 + n_2 - 2$ ozodlik darajali St'yudent taqsimotiga ega bo'ladi. St'yudent taqsimoti $n_1 + n_2$ parametr bilan aniqlanishini, μ va σ parametrlarga bog'liq emasligini osongina ko'rish mumkin. Demak,

$$P\{t_{\alpha/2}(n_1 + n_2 - 2) < T < t_{1-\alpha/2}(n_1 + n_2 - 2)\} =$$
$$= P\{t_{\alpha/2}(n_1 + n_2 - 2) < \frac{\overline{X} - \mu}{S}\sqrt{n_1 + n_2} < t_{1-\alpha/2}(n_1 + n_2 - 2)\} = 1 - \alpha$$

bu yerda $t_{\alpha/2}(n_1 + n_2 - 2)$ va $t_{1-\alpha/2}(n_1 + n_2 - 2)$ lar $k = n_1 + n_2 - 2$ ozodlik darajali St' yudent taqsimotning kvantillari.

Qavis ichidagi

$$t_{\alpha/2}(n_1 + n_2 - 2) < \frac{\overline{X} - \mu}{S}\sqrt{n_1 + n_2} < t_{1-\alpha/2}(n_1 + n_2 - 2)$$

tengsizlikni μ ga nisbatan yechib,

$$\overline{X} - \frac{S}{\sqrt{n_1 + n_2}}t_{1-\alpha/2}(n_1 + n_2 - 2) < \mu < \overline{X} - \frac{S}{\sqrt{n_1 + n_2}}t_{\alpha/2}(n_1 + n_2 - 2)$$

ni hosil qilamiz.

St'yudent taqsimotining kvantillari $t_{\alpha/2}(n_1 + n_2 - 2) = -t_{1-\alpha/2}(n_1 + n_2 - 2)$ day bog'lanishdaligini hisobga olib, berilgan $1-\alpha$ ishonchlilik ehtimolida μ uchun ishonchli intervalni ,ya'ni

$$\overline{X} - \frac{S}{\sqrt{n_1 + n_2}} t_{1-\alpha/2}(n_1 + n_2 - 2) < \mu < \overline{X} + \frac{S}{\sqrt{n_1 + n_2}} t_{1-\alpha/2}(n_1 + n_2 - 2)$$

ni hosil qilamiz.

Adabiyotlar

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