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KO'P KOMPLEKS O'ZGARUVCHILI KOSHI TIPIDAGI INTEGRALNING MAVJUDLIK SHARTI

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ARTICLEINFO.

Kalit so`zlar: Mavjudlik Sharti.

Annotatsiya

Koshi tipidagi integralning ko'p kompleks o'zgaruvchili sohada mavjudlik sharti haqida.

$\varphi(t) = \varphi(t_1, t_2, \dots, t_n)$ funksiya $\Delta -$ ostovda aniqlangan bo'lsin.

$t = (t_1, t_2, \dots, t_n) \in \Delta$ nuqtani markaz qilib istalgancha kichik r_k radiusli $C(t_k, r_k)$ polisilindir chizaylikki, har bir $D_k^{\cdot}$ kontur polisilindirlar bilan faqat ikki nuqtada kesishsin. $D_k^{\cdot}$ konturning $C(t_k, r_k)$ polisilindirning ichkarisidagi qismini $d_k^{\cdot}$ qolgan qismini esa $D_k^{\cdot} - d_k^{\cdot}$ bilan belgilaymiz. $D_\varepsilon = (D_1^{\cdot} - d_k^{\cdot}) \times (D_2^{\cdot} - d_k^{\cdot}) \times \dots \times (D_n^{\cdot} - d_k^{\cdot})$ deb belgilaymiz. Ravshanki, $\varepsilon \to 0$ da $D_\varepsilon \to \Delta$.

1-ta'rif. Agar $\lim_{\varepsilon \to 0} \Phi_\varepsilon(t)$ (bunda

$$\Phi_\varepsilon(t) = \frac{1}{(2\pi i)^n} \int_{D_\varepsilon} \frac{\varphi(\tau)}{\prod_{k=1}^{n} (\tau_k - z_k)} d\tau) \qquad (1)$$

limit mavjud bo'lsa, u holda

$$\Phi(t) = \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\varphi(\tau)}{\prod_{k=1}^{n} (\tau_k - t_k)} d\tau \qquad (2)$$

maxsus integral Koshining bosh qiymat manosida mavjud deyiladi va u

$$\lim_{\varepsilon \to 0} \Phi_\varepsilon(t) = V \cdot P \frac{1}{(2\pi i)^n} \int_{\Delta} \frac{\varphi(\tau)}{\prod_{k=1}^{n} (\tau_k - t_k)} d\tau = V \cdot P\, \Phi(t)$$

kabi belgilanadi.

Bundan keyin (2) maxsus integralning bosh qismini oddiy integral deb ataymiz va uni qisqacha $\Phi(t) = \frac{1}{2^n} S\varphi(\tau)$ deb ham belgilaymiz.

(2) maxsus integralning bosh qiymati har doim ham mavjud bo'lavermaydi. Shuning uchun u

qanday funksiyalar sinfi uchun o'rinli bo'ladi degan masala bilan shug'ullanamiz.

1-teorema. Agar (2) maxsus integralning zichligi Δ ostovda Gyolder shartini qanoatlantirsa, u holda (2) maxsus integral bosh qiymat ma'nosida mavjud bo'ladi.

Isbot. Yozishni qisqartirish maqsadida teoremani $n=3$ bo'lganda isbotlaymiz. Teoremani isbot qilish jarayonida Gyolder shartini qanoatlantiruvchi tengsizliklardan foydalanamiz.

(2) integralning zichligi $\varphi(\tau_1,\tau_2,\tau_3)$ ni

$$\varphi(\tau_1,\tau_2,\tau_3)=\varphi_3(\tau;t)+\varphi_3(\tau_{t_1};t)+\varphi_3(\tau_{t_2};t)+\varphi_3(\tau_{t_3};t)++\varphi_3(t_{\tau_1};t)+$$

$+\varphi_3(t_{\tau_2};t)+\varphi_3(t_{\tau_3};t)+\varphi(t_1,t_2,t_3)$ ayniyatning o'ng tomoni bilan almashtirib topamiz.

$$\Phi_\varepsilon(t)=\frac{1}{(2\pi i)^3}\int_{D_\varepsilon}\frac{\varphi_3(\tau,t)}{\prod_{k=1}^3(\tau_k-t_k)}d\tau+\frac{1}{(2\pi i)^3}\sum_{k=1}^3\int_{D_\varepsilon}\frac{\varphi_3(\tau_{t_k},t)}{\prod_{k=1}^3(\tau_k-t_k)}d\tau+$$

$$+\frac{1}{(2\pi i)^3}\sum_{k=1}^3\int_{D_\varepsilon}\frac{\varphi_3(t_{\tau_k},t)}{\prod_{k=1}^3(\tau_k-t_k)}d\tau+\frac{\varphi(t)}{(2\pi i)^3}\int_{D_\varepsilon}\frac{1}{\prod_{k=1}^3(\tau_k-t_k)}d\tau \qquad (3)$$

(3) ning o'ng tomonidagi integralni mos ravishda

$$\overset{3}{S}_\varepsilon\varphi_3(\tau,t);\quad \overset{2}{S}_\varepsilon\varphi_3(\tau_{t_k},t)\ (k=1,2,3);\quad \overset{1}{S}_\varepsilon\varphi_3(t_{\tau_k},t)\ (k=1,2,3)$$

va $\overset{0}{S}_\varepsilon\cdot\ \overset{3}{S}_\varepsilon\varphi_3(\tau,t)$ ni baholash uchun $|\varphi_3(\tau;t)|\le 4\prod_{k=1}^3 A_k^{\frac{1}{3}}|\tau_k-t_k|^{\frac{\alpha_k}{3}}$ dan foydalanib topamiz.

$$\left|\overset{3}{S}_\varepsilon\varphi_3(\tau,t)\right|\le\frac{4\sqrt[3]{A_1A_2A_3}}{(2\pi)^3}\int_{D_\varepsilon}\prod_{k=1}^3|\tau_k-t_k|^{\frac{\alpha_k}{3}-1}|d\tau_k|=$$

$$=\frac{1}{2\pi^3}\prod_{k=1}^3\sqrt[3]{A_k}\int_{D_k^{\cdot}-d_k^{\cdot}}|\tau_k-t_k|^{\frac{\alpha_k}{3}-1}|d\tau_k| \qquad (4)$$

(4) ning o'ng tomonidagi ko'paytmaning tagida turgan integrallarning har biri ε→0 da oddiy Riman ma'nosida mavjud. Qolgan integrallarni baholashda quydagi tengsizliklardan foydalanamiz.

$$\left|\frac{1}{2\pi i}\int_{D_k^{\cdot}-d_k^{\cdot}}\frac{d\tau_k}{\tau_k-t_k}\right|<1,\qquad k=1,2,3 \qquad (5)$$

$\overset{3}{S}_\varepsilon\varphi_3(\tau_{t_3},t)$ ni baholashda (5) va

$$\left|\varphi_3(t_{\tau_p};t)\right|\le 2\prod_{k=1}^3 A_k^{\frac{1}{2}}|\tau_k-t_k|^{\frac{\alpha_k}{2}};\ p=\overline{1,3};\qquad k\ne p,$$

dan $p=3$ da foydalanib topamiz.

$$\left|\overset{3}{S}_{\varepsilon}\varphi_3(\tau_{t_3},t)\right| = \left|\frac{1}{2\pi i}\int_{D_k^{\cdot}-d_3^{\cdot}}\frac{d\tau_3}{\tau_3-t_3}\right|\left|\frac{1}{(2\pi i)^2}\int_{D_k^{\cdot}-d_1^{\cdot}}\int_{D_k^{\cdot}+d_2^{\cdot}}\frac{\varphi_3(\tau_{t_3},t)d\tau_1 d\tau_2}{(\tau_1-t_1)(\tau_2-t_2)}\right|$$

$$\leq \frac{1}{2\pi^2}\prod_{k=1}^{2}\sqrt{A_k}\int_{D_k^{\cdot}-d_k^{\cdot}}|\tau_k-t_k|^{\frac{\alpha_k}{2}-1}|d\tau_k| \qquad (6)$$

(6) ning o’ng tomonidagi ko’paytmaning tagida turgan har biri ε→0 da oddiy Riman ma’nosida mavjud bo’ladi. Xuddi shunday $\overset{2}{S}_{\varepsilon}\varphi_3(\tau_{t_2},t)$ va $\overset{2}{S}_{\varepsilon}\varphi_3(\tau_{t_1},t)$ integrallar uchun ham baholar o’rinli bo’ladi.

$\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_3},t)$ uchun (5) ni va $\left|\varphi_3(t_{\tau_p},t)\right| \leq A_p|\tau_p-t_p|^{\alpha_p},\ p=\overline{1,3}$ ni $p=3$ da e’tiborga olgan holda topamiz.

$$\left|\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_3},t)\right| = \left|\frac{1}{2\pi i}\int_{D_1^{\cdot}-d_1^{\cdot}}\frac{d\tau_1}{\tau_1-t_1}\right|\cdot\left|\frac{1}{2\pi i}\int_{D_2^{\cdot}-d_2^{\cdot}}\frac{d\tau_2}{\tau_2-t_2}\right|\cdot$$

$$\cdot\left|\frac{1}{2\pi i}\int_{D_3^{\cdot}-d_3^{\cdot}}\frac{\varphi_3(t_{\tau_3},t)d\tau_3}{\tau_3-t_3}\right| \leq \frac{A_3}{2\pi}\int_{D_3^{\cdot}-d_3^{\cdot}}|\tau_3-t_3|^{\alpha_3-1}|d\tau_3|.$$

Bundan ε→0 da $\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_3},t)$ ning yaqinlashuvchiligi kelib chiqadi.

Xuddi shunday $\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_2},t)$va $\overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_1},t)$ larning yaqinlashuvchiligini ko’rish qiyin emas. Ma’lumki,

$$\frac{1}{2\pi i}\int_{D_k^{\cdot}}\frac{d\tau_k}{\tau_k-z_k} = \begin{cases}1, & z_k\in D_k^{+}\\ \frac{1}{2}, & z_k\in D_k^{\cdot}\\ 0, & z_k\in D_k^{-}\end{cases} \qquad (7)$$

(7) e’tiborga olgan holda ε→0 da $\overset{0}{S}_{\varepsilon}\to\frac{1}{8}$ ga intilishi kelib chiqadi.

$\overset{3}{S}_{\varepsilon}\varphi_3(\tau,t);\ \overset{2}{S}_{\varepsilon}\varphi_3(\tau_{t_k},t)\ (k=1,2,3);\ \overset{1}{S}_{\varepsilon}\varphi_3(t_{\tau_k},t)\ (k=1,2,3)$ larni ε→0 da mos ravishda

$$\overset{3}{\psi}_{?}(t),\ \frac{1}{2}\overset{2}{\psi}_1(t),\ \frac{1}{2}\overset{2}{\psi}_2(t),\ \frac{1}{2}\overset{2}{\psi}_3(t),\ \frac{1}{4}\overset{1}{\psi}_1(t),\ \frac{1}{4}\overset{1}{\psi}_2(t),\ \frac{1}{4}\overset{1}{\psi}_3(t),$$

deb belgilaymiz, bunda misol uchun

$$\overset{3}{\psi}_{?}(t_1,t_2,t_3) = \frac{1}{(2\pi i)^3}\int\frac{\varphi_3(\tau,t)}{\prod^3(\tau-t)}d\tau \qquad (8)$$

$$\overset{2}{\psi}_{1 ?}(t_1,t_2,t_3)=\frac{1}{(2\pi i)^2}\int\int\frac{\varphi_3(\tau_{t_1},t)}{(\tau_2-t_2)(\tau_3-t_3)}d\tau_2 d\tau_3 \qquad (9)$$

$$\overset{1}{\psi}_{1 ?}(t_1,t_2,t_3)=\frac{1}{2\pi i}\int\frac{\varphi_3(t_{\tau_1},t)}{\tau_1-t_1}d\tau_1 \qquad (10).$$

Shunday qilib $n=3$ bo’lganda (2) integralning bosh qiymati:

$$\Phi(t_1,t_2,t_3)=\overset{3}{\psi}(t_1,t_2,t_3)+\frac{1}{2}\overset{2}{\psi}_1(t_1,t_2,t_3)+\frac{1}{2}\overset{2}{\psi}_2(t_1,t_2,t_3)+$$
$$+\frac{1}{2}\overset{2}{\psi}_3(t_1,t_2,t_3)+\frac{1}{4}\overset{1}{\psi}_1(t_1,t_2,t_3)+\frac{1}{4}\overset{1}{\psi}_2(t_1,t_2,t_3)+\frac{1}{4}\overset{1}{\psi}_3(t_1,t_2,t_3)+$$
$$+\frac{1}{8}\varphi(t_1,t_2,t_3) \qquad (11)$$

bo’ladi.

Foydalanilgan adabiyotlar.

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