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GYOLDER SINIFI

O'razaliyev Shirinboy Bo'ron o'g'li

Samarqand iqtisodiyot va servis instituti, shirinboy.urazaliyev@mail.ru

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Annotatsiya

$\varphi(t_1, t_2, \dots, t_n)$ funksiyaning ko'p o'zgaruvchili kompleks tekislikda Gyolder shartini qanoatlantirishi haqida.

$\varphi(t)$ funksiya Δ-ostovda aniqlangan bo'lsin.

1-ta'rif. Agar $\varphi(t) = \varphi(t_1, t_2, \dots, t_n)$ funksiya

$t = (t_1, t_2, \dots, t_n), \tau = (\tau_1, \tau_2, \dots, \tau_n) \in \Delta$ nuqtalar uchun

$$|\varphi(t) - \varphi(\tau)| \leq \sum_{k=1}^{n} A_k |t_k - \tau_k|^{\alpha_k} \qquad (1)$$

tengsizlikni qanoatlantirsa, u holda $\varphi(t)$ funksiya Δ da Gyolder shartini qanoatlantiradi deyiladi, bunda A_k $(k = \overline{1,n})$ musbat sonlar Gyolder o'zgarmasi, α_k esa $0 < \alpha_k \leq 1$ $(k = \overline{1,n})$ tengsizlikni qanoatlantiruvchi o'zgarmas sonlar bo'lib, u Gyolder ko'rsatkichi deyilada.

Gyolder shartini qanoatlantiruvchi funksiyalar sinfini $H_{\alpha_k}(\Delta)$ deb belgilaymiz.

Agar $\varphi(t) \in H_{\alpha_k}(\Delta)$ bo'lsa, u holda $\varphi(t) \in H_{\beta_k}(\Delta)$ $(\beta_k < \alpha_k,\ k = \overline{1,n})$ bo'ladi. Haqiqatdan ham shartga ko'ra $\varphi(t) \in H_{\alpha_k}(\Delta)$;

$$|\varphi(t) - \varphi(\tau)| \leq \sum_{k=1}^{n} A_k |t_k - \tau_k|^{\alpha_k}$$

bundan $\sum_{k=1}^{n} A_k |t_k - \tau_k|^{\alpha_k} = \sum_{k=1}^{n} A_k |t_k - \tau_k|^{\alpha_k - \beta_k} \cdot |t_k - \tau_k|^{\beta_k} \leq$

$\leq \sum_{k=1}^{n} \check{A}_k |t_k - \tau_k|^{\beta_k} \quad \Rightarrow \varphi(t) \in H_{\beta_k}(\Delta)$ ekanligi kelib chiqadi.

1-teorema. $\varphi(t)$ funksiya Δ da (1) Gyolder shartini qanoatlantirishi

uchun har bir t_p argumenti bo'yicha boshqa t_k $(p \neq k, k = \overline{1,n})$

argumentlariga nisbatan tekis Gyolder shartini qanoatlantirishi zarur va yetarli.

Isbot. Haqiqatdan ham $\tau_k = t_k, k = \overline{1,n}, p \neq k$ bo'lganda (1) dan

$$\left|\varphi(t) - \varphi(t_{\tau_p})\right| \leq A_p\left|t_p - \tau_p\right|^{\alpha_p}, \quad p = \overline{1,n} \qquad (2)$$

o'rinli bo'ladi. Xuddi shunday quydagi tengsizliklarni isbotlash qiyin emas:

$$\begin{cases} \left|\varphi\left(t_{\tau_p}\right) - \varphi(t_{\tau_{pq}})\right| \leq A_q\left|t_q - \tau_q\right|^{\alpha_q}, \quad p,q = \overline{1,n}, p < q \\ \left|\varphi\left(t_{\tau_{pq}}\right) - \varphi(t_{\tau_{pqr}})\right| \leq A_r\left|t_r - \tau_r\right|^{\alpha_r}, \quad p,q,r = \overline{1,n}, p < q < r \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \left|\varphi\left(\tau_{t_p}\right) - \varphi(\tau_{t_{pq}})\right| \leq A_q\left|t_q - \tau_q\right|^{\alpha_q}, \quad p,q = \overline{1,n}, p < q \\ \left|\varphi(\tau) - \varphi(\tau_{t_p})\right| \leq A_p\left|t_p - \tau_p\right|^{\alpha_p}, \quad p = \overline{1,n} \end{cases} \qquad (3)$$

(3) tengsizliklar sistemasini e'tiborga olgan holda ushbu

$$|\varphi(t) - \varphi(\tau)| \leq \left|\varphi(t) - \varphi\left(t_{\tau_1}\right)\right| + \left|\varphi\left(t_{\tau_1}\right) - \varphi\left(t_{\tau_{12}}\right)\right| + \cdots +$$
$$+\left|\varphi\left(\tau_{t_{n-1,n}}\right) - \varphi\left(\tau_{t_n}\right)\right| + \left|\varphi\left(\tau_{t_n}\right) - \varphi(\tau)\right|,$$

tengsizlikdan teoremaning tasdiqi kelib chiqadi.

Keyinchalik ishlatiladigan ba'zi yig'indilar.

$$\varphi_n(\tau;t) = \varphi_n(\tau_1, \tau_2, \dots, \tau_n; t_1, t_2, \dots, t_n)$$

orqali quydagi yig'indini belgilaymiz

$$\varphi_n(\tau;t) = \varphi(\tau) - \sum_{p=1}^{n}\varphi\left(\tau_{t_p}\right) + \sum_{p=1}^{n}\sum_{q=1}^{n}\varphi\left(\tau_{t_{pq}}\right) - \cdots +$$
$$+(-1)^{n-2}\sum_{p=1}^{n}\sum_{q=1}^{n}\varphi\left(t_{\tau_{pq}}\right) + (-1)^{n-1}\sum_{p=1}^{n}\varphi\left(t_{\tau_p}\right) + (-1)^n\varphi(t), p < q \qquad (4)$$

(4) dan

$$\varphi_n(\tau;t) = (-1)^n\varphi_n(t;\tau) \qquad (5)$$

tenglikning to'g'riligini tekshirib ko'rish qiyin emas.

$\varphi_n(\tau;t)$ yig'indi 2^n ta qo'shiluvchiga ega.

Masalan, $n = 3$ bo'lganda

$$\varphi_3(\tau;t) = \varphi(\tau_1,\tau_2,\tau_3) - \varphi(\tau_1,\tau_2,t_3) - \varphi(\tau_1,t_2,\tau_3) - \varphi(t_1,\tau_2,\tau_3) +$$
$$+\varphi(\tau_1,t_2,t_3) + \varphi(t_1,\tau_2,t_3) + \varphi(t_1,t_2,\tau_3) - \varphi(t_1,t_2,t_3) \qquad (6)$$

$$\varphi_n\left(\tau_{t_p};t\right) = \varphi_n(\tau_1,\tau_2,\dots,\tau_{p-1},t_p,\tau_{p+1},\dots,\tau_n;t_1,t_2,\dots,t_n)$$

orqali ushbu

$$\varphi_n\left(\tau_{t_p};t\right)=\varphi\left(\tau_{t_p}\right)-\sum_{\substack{q=1\\q\neq p}}^{n}\varphi\left(\tau_{t_{pq}}\right)+\sum_{\substack{q=1\\q,r\neq p}}^{n}\sum_{\substack{r=1\\q<r}}^{n}\varphi\left(\tau_{t_{pqr}}\right)-\cdots+$$

$$+(-1)^{n-3}\sum_{\substack{q=1\\q,r\neq p}}^{n}\sum_{\substack{r=1\\q<r}}^{n}\varphi\left(t_{\tau_{qr}}\right)+(-1)^{n-2}\sum_{\substack{q=1\\q\neq p}}^{n}\varphi\left(t_{\tau_q}\right)+(-1)^{n+1}\varphi(t) \qquad 7)$$

(7) yig'indi 2^{n-1} ta qo'shiluvchiga ega va u (6) ga o'xshash tuziladi, lekin undan bitta qo'shiluvchi kam.

$n=3$ bo'lganda (7) ning ko'rinishi quydagi ko'rinishga keladi:

$$\begin{cases}\varphi_3\left(\tau_{t_3};t\right)=\varphi_3\left(t_{\tau_{12}};t\right)=\varphi(\tau_1,\tau_2,t_3)-\varphi(\tau_1,t_2,t_3)-\varphi(t_1,\tau_2,\tau_3)+\varphi(\tau_1,\tau_2,t_3)\\ \varphi_3\left(\tau_{t_2};t\right)=\varphi_3\left(t_{\tau_{13}};t\right)=\varphi(\tau_1,t_2,\tau_3)-\varphi(\tau_1,t_2,t_3)-\varphi(t_1,t_2,\tau_3)+\varphi(t_1,t_2,t_3)\\ \varphi_3\left(\tau_{t_1};t\right)=\varphi_3\left(t_{\tau_{23}};t\right)=\varphi(t_1,\tau_2,\tau_3)-\varphi(t_1,t_2,\tau_3)-\varphi(t_1,\tau_2,t_3)+\varphi(t_1,t_2,t_3)\end{cases} \quad (8)$$

Xuddi shunday $\varphi_n\left(\tau_{t_{pq}};t\right)$, $\varphi_n\left(\tau_{t_{pqr}};t\right),\ldots,\varphi_n\left(t_{\tau_{pq}};t\right)$ va

$\varphi_n\left(t_{\tau_p};t\right)$ va hokazo yig'indilar uchun ham tuziladi.

$n=3$ bo'lganda bu yig'indilarning ko'rinish quydagicha bo'ladi:

$$\begin{cases}\varphi_3\left(\tau_{t_{12}};t\right)=\varphi_3\left(t_{\tau_3};t\right)=\varphi(t_1,t_2,\tau_3)-\varphi(t_1,t_2,t_3)\\ \varphi_3\left(\tau_{t_{13}};t\right)=\varphi_3\left(t_{\tau_2};t\right)=\varphi(t_1,\tau_2,t_3)-\varphi(t_1,t_2,t_3)\\ \varphi_3\left(\tau_{t_{23}};t\right)=\varphi_3\left(t_{\tau_1};t\right)=\varphi(\tau_1,t_2,t_3)-\varphi(t_1,t_2,t_3)\end{cases} \qquad (9)$$

(6), yuqoridagi ikkita sistema va $\varphi(t_1,t_2,t_3)$ lar qo'shish natijasida ushbu

$$\varphi(\tau_1,\tau_2,\tau_3)=\varphi_3(\tau;t)+\varphi_3\left(\tau_{t_1};t\right)+\varphi_3\left(\tau_{t_2};t\right)+\varphi_3\left(\tau_{t_3};t\right)++\varphi_3\left(t_{\tau_1};t\right)+$$

$$+\varphi_3\left(t_{\tau_2};t\right)+\varphi_3\left(t_{\tau_3};t\right)+\varphi(t_1,t_2,t_3) \qquad (10)$$

ayniyatni hosil qilamiz. Umumiy holda bu ayniyatning ko'rinishi quydagicha bo'ladi.

$$\varphi(\tau)=\varphi_n(\tau;t)+\sum_{p=1}^{n}\varphi_n(\tau_{t_p};t)+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\varphi_n\left(\tau_{t_{pq}};t\right)+\cdots+$$

$$+\sum_{\substack{p=1\\p<q}}^{n}\sum_{q=1}^{n}\varphi_n\left(t_{\tau_{pq}};t\right)+\sum_{p=1}^{n}\varphi_n(t_{\tau_p};t)+\varphi(t) \qquad (11)$$

bu ayniyatning to'g'riligini yuqoridagi singari tekshirib ko'rish qiyin emas.

Qaralayotgan (4) yig'indining hadlarini ikkitadan shunday gruppalarga ajratayliki, har bir gruppa faqat bitta aniq argument bilan farq qiladigan funksiyalar ayirmasi shaklida tasvirlansin. (4) yig'indida shunday gruppalar soni 2^{n-1} ta bo'ladi. Gruppalashlar soni esa aniq n ga teng bo'ladi.

Masalan $n = 3$ bo'lganda (4) yig'indi uchun:

$$\begin{cases} \varphi_3(\tau;t) = [\varphi(\tau_1,\tau_2,\tau_3) - \varphi(\tau_1,\tau_2,t_3)] + [\varphi(\tau_1,t_2,t_3) - \varphi(\tau_1,t_2,\tau_3)] \\ +[\varphi(t_1,\tau_2,t_3) - \varphi(t_1,\tau_2,\tau_3)] + [\varphi(t_1,t_2,\tau_3) - \varphi(t_1,t_2,t_3)], \\ \varphi_3(\tau;t) = [\varphi(\tau_1,\tau_2,\tau_3) - \varphi(\tau_1,t_2,\tau_3)] + [\varphi(\tau_1,t_2,t_3) - \varphi(\tau_1,\tau_2,t_3)] \\ +[\varphi(t_1,\tau_2,t_3) - \varphi(t_1,t_2,t_3)] + [\varphi(t_1,t_2,\tau_3) - \varphi(t_1,\tau_2,\tau_3)], \\ \varphi_3(\tau;t) = [\varphi(\tau_1,\tau_2,\tau_3) - \varphi(t_1,\tau_2,\tau_3)] + [\varphi(\tau_1,t_2,t_3) - \varphi(t_1,t_2,t_3)] \\ +[\varphi(t_1,\tau_2,t_3) - \varphi(\tau_1,\tau_2,t_3)] + [\varphi(t_1,t_2,\tau_3) - \varphi(\tau_1,t_2,\tau_3)]. \end{cases}$$

Xuddi shunday mulohazalarni $\varphi_n\left(\tau_{t_p};t\right)$, $\varphi_n\left(\tau_{t_{pq}};t\right), \ldots, \varphi_n\left(t_{\tau_{pq}};t\right)$ va $\varphi_n\left(t_{\tau_p};t\right)$ yig'indilarga ham qo'llash mumkin.

Bu punktda (4) da qaralgan yig'indilarni baholaymiz; buning uchun avvalo $n = 3$ uchun qaraymiz. Yuqoridagi sistemadan yig'indining moduli, modullar yig'indisidan kichik yoki tengligini e'tiborga olgan holda (4) dan topamiz:

$$|\varphi_3(\tau;t)| \le 4A_k|\tau_k - t_k|^{\alpha_k}, \quad k = 1,2,3.$$

osonlik bilan ko'rish mumkinki, umumiy holda:

$$\begin{cases} |\varphi_n(\tau;t)| \le 2^{n-1}A_k|\tau_k - t_k|^{\alpha_k}, \quad k = \overline{1,n} \\ \left|\varphi_n\left(\tau_{t_p};t\right)\right| \le 2^{n-2}A_k|\tau_k - t_k|^{\alpha_k}, \quad k \ne p; \ k,p = \overline{1,n} \\ \left|\varphi_n\left(\tau_{t_{pq}};t\right)\right| \le 2^{n-3}A_k|\tau_k - t_k|^{\alpha_k}, \quad k \ne p,q; \ p < q; \quad k,p,q = \overline{1,n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \left|\varphi_n\left(t_{\tau_{pqr}};t\right)\right| \le 2^2A_k|\tau_k - t_k|^{\alpha_k}, \quad p < q < r; \ k \ne p,q,r; \quad k,p,q,r = \overline{1,n} \\ \left|\varphi_n\left(t_{\tau_{pq}};t\right)\right| \le 2A_k|\tau_k - t_k|^{\alpha_k}, \quad k \ne p,q; \ p < q; \quad k,p,q = \overline{1,n} \end{cases} \tag{12}$$

bu birinchisidan foydalanib topamiz:

$$|\varphi_n(\tau;t)| \le [|\varphi_n(\tau;t)|^n]^{\frac{1}{n}} \le \left[\prod_{k=1}^{n} 2^{n-1}A_k\,|\tau_k - t_k|^{\alpha_k}\right]^{\frac{1}{n}} =$$

$$= 2^{n-1}\prod_{k=1}^{n} A_k^{\frac{1}{n}}|\tau_k - t_k|^{\frac{\alpha_k}{n}}$$

Xuddi shunday mulohazalardan keyin yuqoridagi sistemaning qolgan tensizliklari uchun ham quydagi tengsizliklarni olish mumkin:

$$\left|\varphi_n\left(t_{\tau_{pq}};t\right)\right| \le 2\sqrt{A_p \cdot A_q}\,\left|\tau_p - t_p\right|^{\frac{\alpha_p}{2}} \cdot \left|\tau_q - t_q\right|^{\frac{\alpha_q}{2}};$$

$$p,q = \overline{1,n}; \ p < q, \tag{13}$$

$$\left|\varphi_n\left(t_{\tau_{pqr}};t\right)\right| \le 4\sqrt[3]{A_p \cdot A_q \cdot A_r}\,\left|\tau_p - t_p\right|^{\frac{\alpha_p}{3}} \cdot \left|\tau_q - t_q\right|^{\frac{\alpha_q}{3}} \cdot \left|\tau_r - t_r\right|^{\frac{\alpha_r}{3}};$$

$$p < q < r; \;\; p,q,r = \overline{1,n}; \qquad (14)$$

… … … … … … … … … … … … … … …

$$\left|\varphi_n\left(\tau_{t_{pq}};t\right)\right| \leq 2^{n-3}\prod_{k=1}^{n} A_k^{\frac{1}{n-2}}\left|\tau_p - t_p\right|^{\frac{\alpha_k}{n-2}};$$

$$p,q = \overline{1,n}; \quad p < q \quad k \neq p,q \qquad (15)$$

$$\left|\varphi_n\left(\tau_{t_p};t\right)\right| \leq 2^{n-2}\prod_{k=1}^{n} A_k^{\frac{1}{n-1}}\left|\tau_k - t_k\right|^{\frac{\alpha_k}{n-1}}; \quad p = \overline{1,n}; \quad k \neq p, \qquad (16)$$

$$|\varphi_n(\tau;t)| \leq 2^{n-1}\prod_{k=1}^{n} A_k^{\frac{1}{n}}\left|\tau_k - t_k\right|^{\frac{\alpha_k}{n}}; \qquad (17)$$

(3) ning birinchisini $\varphi_n\left(\tau_{t_p};t\right)$ yig'indi orqali quydagicha yozamiz:

$$\left|\varphi_n\left(t_{\tau_p},t\right)\right| \leq A_p\left|\tau_p - t_p\right|^{\alpha_p}, \quad p = \overline{1,n} \qquad (18)$$

takidlaymizki, $n = 2, 3$ hollar uchun ham xuddi shunday yig'indilar uchun baholarni olamiz:

$$\left|\varphi_2\left(t_{\tau_p},t\right)\right| \leq A_p\left|\tau_p - t_p\right|^{\alpha_p}, \quad p = \overline{1,2} \qquad (19)$$

$$|\varphi_2(\tau;t)| \leq 2\sqrt{A_1 \cdot A_2}\,\left|\tau_1 - t_1\right|^{\frac{\alpha_1}{2}} \cdot \left|\tau_2 - t_2\right|^{\frac{\alpha_2}{2}}, \quad (20)$$

$$\left|\varphi_3\left(t_{\tau_p},t\right)\right| \leq A_p\left|\tau_p - t_p\right|^{\alpha_p}, \quad p = \overline{1,3} \qquad (21)$$

$$\left|\varphi_3\left(t_{\tau_p};t\right)\right| \leq 2\prod_{k=1}^{3} A_k^{\frac{1}{2}}\left|\tau_k - t_k\right|^{\frac{\alpha_k}{2}}; \quad p = \overline{1,3}; \quad k \neq p, \quad (22)$$

$$|\varphi_3(\tau;t)| \leq 4\prod_{k=1}^{3} A_k^{\frac{1}{3}}\left|\tau_k - t_k\right|^{\frac{\alpha_k}{3}}; \qquad (23)$$

Foydalanilgan adabiyotlar.

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