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KOMPLEKS SONNING GEOMETRIK TASVIRI MAPLE TIZIMIDA

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Kalit so'zlar:

Kompleks son, haqiqiy son, mavhum birlik, algebraik shakl, mavhum son, qo'shma kompleks son, qarama-qarshi kompleks son, modul, argument, trigonometrik shakl.

Annotatsiya:

Maqolada kompleks sonlarning matematik apparati tasvirlangan: asosiy tushunchalar va ta'riflar berilgan, kompleks sonlarning algebraik va trigonometrik shakllarda aniqlash hamda geometrik talqini bo'yicha masalalarni yechimlari Maple tizimida bajarilgan[1,2,3,7,8,9]..

KIRISH: Hozirgi vaqtda har qanday jiddiy hisob-kitoblar, qoida tariqasida, kompyuterlarda va birinchi navbatda, shaxsiy kompyuterlarda amalga oshiriladi. Ushbu maqolada Maple dasturidan foydalanib kompleks sonlar uchun tuzilgan matematik modellarning sifati va undan raqamli usullarda foydalanib tahlil va qaror qabul qilishda axamiyatli ekanligi ko'rsatilgan

1.Asosiy ta'riflar.

1-ta'rif. *a* va *b –haqiqiy sonlar uchun yozilgan* $z = a + ib$ ko'rinishidagi ifodaga *kompleks son* deb aytiladi.

Bunda $i = \sqrt{-1}$ $(i^2 = -1)$ tenglik bilan aniqlanuvchi *mavhum birlik* deb ataluvchi birlik. *z* kompleks sonning haqiqiy va mavhum qismlari quyidagicha belgilanadi:

$$\operatorname{Re} z = a, \quad \operatorname{Im} z = b.$$

Xususiy holda, agar $a = 0$ bo‘lsa, u holda $z = 0 + i\,b$ sonni *sof mavhum* son, agar $b = 0$ bo‘lsa, u holda $z = a + i \cdot 0 = a,$ ya’ni haqiqiy son hosil bo‘ladi. Shunday qilib, haqiqiy va mavhum sonlar z kompleks sonlarning xususiy hollaridir.

Kompleks sonning $z = a + bi$ ko‘rinishdagi yozuvi uning *algebraik shakli* deyiladi.

2-ta’rif. Agar ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonlarning haqiqiy qismlari va mavhum qismlari o‘zaro teng bo‘lsa, bu kompleks sonlar *teng*, ya’ni $z_1 = z_2$ bo‘ladi, ($\operatorname{Re} z_1 = \operatorname{Re} z_2$ va $\operatorname{Im} z_1 = \operatorname{Im} z_2$ bo‘lsa, $z_1 = z_2$ hisoblanadi).

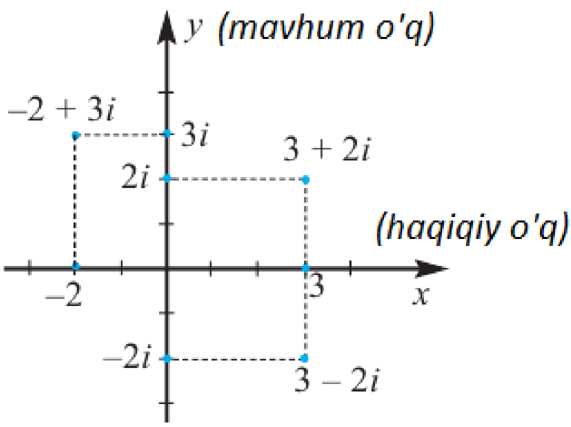
3-ta’rif. $z = a + i\,b$ kompleks sonning haqiqiy va mavhum qismi nolga teng bo‘lsagina, u nolga teng bo‘ladi, ya’ni agar $a = 0$ va $b = 0$ bo‘lsagina $z = 0$, va aksincha.

4-ta’rif. $z = a + i\,b$ va $\bar{z} = a - i\,b$ kompleks sonlar *qo‘shma kompleks sonlar* deyiladi.

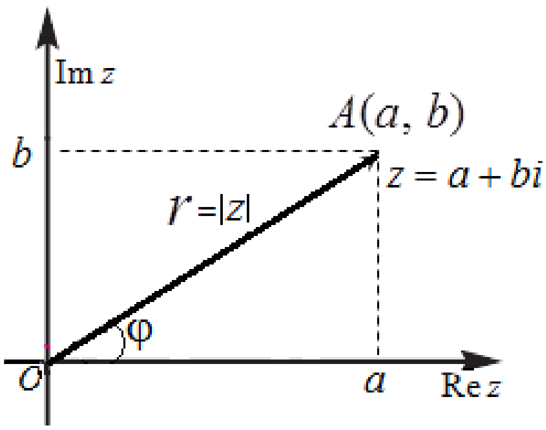
5-ta’rif. $z_1 = a + i\,b$ va $z_2 = -a - i\,b$ kompleks sonlar *qarama-qarshi kompleks sonlar* deyiladi.

2. Kompleks sonlarning geometrik talqini va trigonometrik shakli. Har qanday $z = a + i\,b$ kompleks soni uchun Oxy tekislikda *a* va *b* koordinatali $A(a;b)$ nuqta mos keladi. Bu nuqta kompleks sonning *affeksi* diyiladi. Tekislikning har bir nuqtasiga kompleks son mos keladi va aksincha. Kompleks tekislikda *z* sonni tasvirlovchi nuqtani z nuqta deb ataymiz (1-rasm). Ox o‘qda yotuvchi nuqtalarga haqiqiy sonlar mos keladi, Oy o‘qda yotuvchi nuqtalar sof mavhum sonlarni tasvirlaydi. Shu sababli Ox o‘q *haqiqiy* o‘q. Oy o‘q *mavhum* o‘q deyiladi.

1-musol. $z_1 = -2 + 3i$, $z_2 = 3 + 2i$, $z_3 = 3 - 2i$ kompleks sonlarning nuqtasini tasvirini quyidagi 1-rasmda ko‘ramiz.



1-rasm.



2-rasm.

Koordinatalar boshini qutb boshi nuqtasi deb, Ox o‘qning musbat yo‘nalishini qutb o‘qi deb kompleks tekislikda koordinatalarning qutb sistemasini kiritamiz. φ va r larni $A(a;b)$ nuqtaning qutb koordinatalari deymiz A nuqtaning qutb radiusi r, ya'ni A nuqtadan qutbgacha, O nuqtagacha bo'lgan masofa z kompleks sonning *moduli* deyiladi va *r* yoki $|z|$ kabi belgilanadi(2-rasim).

$$r = |z| = \sqrt{a^2 + b^2} \tag{1}$$

A nuqtaning qutb burchagi φ ni z kompleks sonning *argumenti* quyidagicha belgilanadi:

$$Argz = arctg\frac{b}{a}$$

Ushbu $a = r\,cos\varphi,\ b = r\sin\varphi$ tengliklarni hisobga olib, z kompleks sonni trigonometrik ko'rinishda ifodalash mumkin:

$$z = a + bi = r(\cos\varphi + i\sin\varphi), \tag{2}$$

bunda $r = |z| = \sqrt{a^2 + b^2}$, $tg\varphi = \frac{b}{a}$.

$$\varphi = \arg z = \begin{cases} arctg\frac{b}{a},\ агар\ a > 0,\ b \geq 0\ бўлса, \\ \pi - arctg\frac{b}{a} +,\ агар\ a < 0\ \ бўлса, \\ arctg\frac{b}{a} + 2\pi,\ агар\ a > 0,\ b < 0\ бўлса. \end{cases} \tag{2'}$$

Yozuvning bu shakli

$$z = r(\cos\varphi + i\sin\varphi) \tag{3}$$

kompleks sonning *trigonometrik shakli* deyiladi.

2-misol. $z = -1 + \sqrt{3}i$ kompleka sonni trigonometrik shaki aniqlang va geometrik tasvirini quring

Yechish: Kompleks sonning *moduli va argumenti* quyidagicha:

$$R = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

$$\text{tg}\ \varphi = \frac{b}{a} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

Bunda φ burchak koordinatalar sistemasining II choragiga tegishli (3-rasm):

$$\varphi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Demak, kompleka sonni trigonometrik shaki va tasviri-grafigi (3a-rasim):

$$z = -1 + \sqrt{3}i = 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$$

Maple dasturi

> restart; a:=-1; b:=sqrt(3); $a := -1 \quad b := \sqrt{3}$

> z:= a+b*I; $z := -1 + I\sqrt{3}$

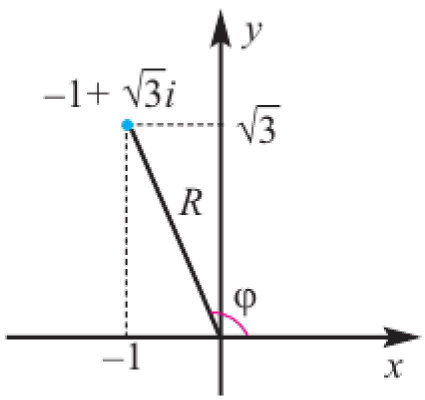
> polar(z); r:=abs(z); phi:=argument(z);

$$\mathrm{polar}\left(2, \frac{2\pi}{3}\right) \quad r := 2 \quad \phi := \frac{2\pi}{3}$$

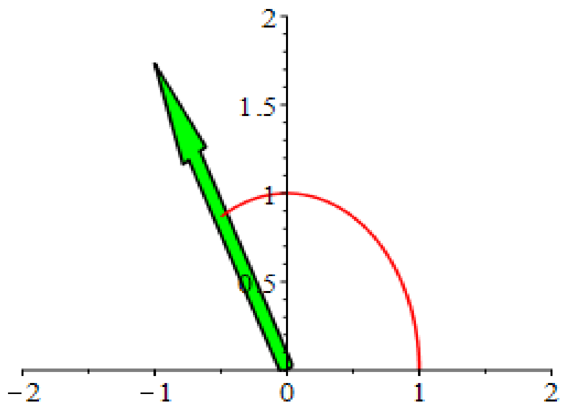
> z:=abs(z)*(Cos(phi)+I*Sin(phi));

$$z := 2\,Cos\left(\frac{2\pi}{3}\right) + 2\,I\,Sin\left(\frac{2\pi}{3}\right)$$

> with(plottools):
L1:=arrow([0,0], [a,b],.1,.2,.3, color=green):
L2:=arc([0,0],1,0..phi,color=red):
plots[display](L1,L2, axes=normal,view=[-r..r, -r..r],scaling=constrained); (3a-rasm)



3-rasm.



3a-rasm.

$z = -1 + \sqrt{3}i$ kompleka sonni algebraik va trigonometrik shaki bo‘yicha qurish dasturini tuzamiz:

Maple dasturi

> restart;

> a:=-1: b:=sqrt(3):

> z:=a+b*I; a:=Re(z); $z := -1 + I\sqrt{3}$

> b:=Im(z); $a := -1 \quad b := \sqrt{3}$

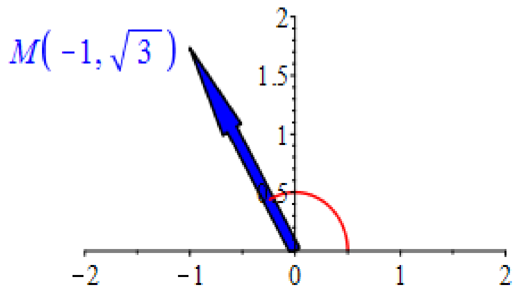
> r:=abs(z); phi:=argument(z); $r := 2 \quad \phi := \frac{2\pi}{3}$

> z1:=abs(z)*(Cos(phi)+I*Sin(phi));

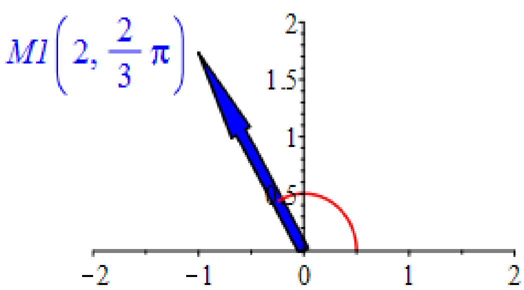
$$z1 := 2\,Cos\left(\frac{2\pi}{3}\right) + 2\,I\,Sin\left(\frac{2\pi}{3}\right)$$

> with(plottools):
Vz:=arrow([0,0], [a,b], .1, .2, .4, color=blue):
> Vza:=arc([0,0],.5,0..arctan(b,a),color=red):

> with(plots): Nuqta:=textplot([[a,b,'M(a,b)']], color=blue,align=Left, font=[TIMES,ROMAN,14]):
> KSA:=plots[display](Vz,Vza,Nuqta, axes=normal, view=[-r..r,0..r], scaling=constrained);(3b-rasm)
> with(plots): Nuqta:=textplot([[r*cos(phi),r*sin(phi),'M1(r,phi)']], color=blue,align=Left, font=[TIMES,ROMAN,14]):
> KST:= plots[display](Vz,Vza, Nuqta, axes=normal, view=[-r..r,0..r],scaling=constrained); (3c-rasm)



3b -rasm.



3c -rasm.

3-misol. Quyidagi kompleks sonlarni trigonometrik shakldа tasvirlang:

$$1)\ \ z=3+\sqrt{3}i;\quad 2)\ \ z=5i;\ \ 3)\ z=-3.$$

Yechish. 1) z kompleks sonning modulini va argumentining qiymatini topamiz. Bu erda $a=3,\ \ b=\sqrt{3}$ ga teng, (1) formulaga ko‘ra topamiz:

$$r=\sqrt{3^2+\left(\sqrt{3}\right)^2}=\sqrt{9+3}=\sqrt{12}=\sqrt{4\cdot 3}=2\sqrt{3}.$$

Argumentini (2’) formuladan $tg\,\varphi=\frac{b}{a}=\frac{\sqrt{3}}{3}$ bo‘lgani uchun, $z=3+\sqrt{3}i$ nuqta I chorakda yotgani uchun $\varphi=\operatorname{arc}tg\frac{\sqrt{3}}{3}=\frac{\pi}{6}$ ni Hosil qilamiz. Bunda (3) formuladan foydalanib, $z=3+\sqrt{3}i$ sonning trigonometrik shaklini Hosil qilamiz:

$$z=3+\sqrt{3}i=2\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right).$$

2) $z=5i$ kompleks sonning modulini va argumentining qiymatini topamiz. Bu yerda $a=0,\ \ b=5$ ga teng, (1) formulaga ko‘ra topamiz: $r=\sqrt{0^2+(5)^2}=\sqrt{0+25}=5.$

Argumentini (2) formuladan $tg\,\varphi=\frac{b}{a}=\frac{5}{0}=\infty$ bo‘lgani uchun, $z=5i$ nuqta I chorak va II chorak o‘rtasida yotgani uchun $\varphi=\operatorname{arc}tg(\frac{5}{0})=arctg(\infty)=\frac{\pi}{2}$ ni hosil qilamiz. Bunda (3) formuladan foydalanib, $z=5i$ sonning trigonometrik shaklini hosil qilamiz: $z=5i=5\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$.

3) $z=-3$ kompleks sonning modulini va argumentining qiymatini topamiz. Bu erda $a=-3,\quad b=0$ ga teng, (1) formulaga ko'ra topamiz: $r=\sqrt{(-3)^2+(0)^2}=\sqrt{9+0}=3$. Argumentini (2) formuladan $tg\,\varphi=\frac{b}{a}=\frac{0}{-3}=0$ bo'lgani uchun, $z=-3$ nuqta II chorak va III chorak o'rtasida yotgani uchun $\varphi=\mathrm{arc}tg(\frac{0}{-3})=arctg(0)=\pi$ ni hosil qilamiz. Bunda (3) formuladan foydalanib, $z=-3$ sonning trigonometrik shaklini hosil qilamiz: $z=-3=3(\cos\pi+i\sin\pi)$.

Misoldagi 1) $z=3+\sqrt{3}i$; 2) $z=5i$; 3) $z=-3$ kompleks sonlarni trigonometrik shaklni aniqlash va geometrik tasvirlash.

M a p l e d a s t u r i

> restart;

> a1:=3: b1:=sqrt(3): z1:=a1+b1*I; $z1 := 3+\mathrm{I}\sqrt{3}$

> polar(z1); r1:=abs(z1); phi1:=argument(z1);

$$\mathrm{polar}\left(2\sqrt{3},\frac{\pi}{6}\right)\qquad r1 := 2\sqrt{3}\qquad \phi1 := \frac{\pi}{6}$$

> z1:=r1*(Cos(phi1)+I*Sin(phi1));

$$z1 := 2\sqrt{3}\left(Cos\left(\frac{\pi}{6}\right)+\mathrm{I}\,Sin\left(\frac{\pi}{6}\right)\right)$$

> a2:=0: b2:=5: z2:=a2+b2*I; $z2 := 5\,\mathrm{I}$

> polar(z2); r2:=abs(z2); phi2:=argument(z2);

$$\mathrm{polar}\left(5,\frac{\pi}{2}\right)\qquad r2 := 5\qquad \phi2 := \frac{\pi}{2}$$

> z2:=r2*(Cos(phi2)+I*Sin(phi2)); #z2:=evalc(%);

$$z2 := 5\,Cos\left(\frac{\pi}{2}\right)+5\,\mathrm{I}\,Sin\left(\frac{\pi}{2}\right)$$

> a3:=-3: b3:=0: z3:=a3+b3*I; $z3 := -3$

> polar(z3); r3:=abs(z3); phi3:=argument(z3);

$$\mathrm{polar}(3,\pi)\qquad r3 := 3\qquad \phi3 := \pi$$

> z3:=r3*(Cos(phi3)+I*Sin(phi3)); $z3 := 3\,Cos(\pi)+3\,\mathrm{I}\,Sin(\pi)$

Kompleks sonlarni vektorlarini qurish:

```
> with(plottools):
L1:= arrow([0,0], [a1,b1], .2, .4, .2, color=green):
L1a:= arc([0,0],1.5,0..arctan(b1/a1),color=green):
L2:= arrow([0,0], [a2,b2], .3, .4, .2, color=blue):
L2a:= arc([0,0],.75,0..1*Pi/2,color=blue):
L3:= arrow([0,0], [a3,b3], .1, .3, .2, color=red):
```

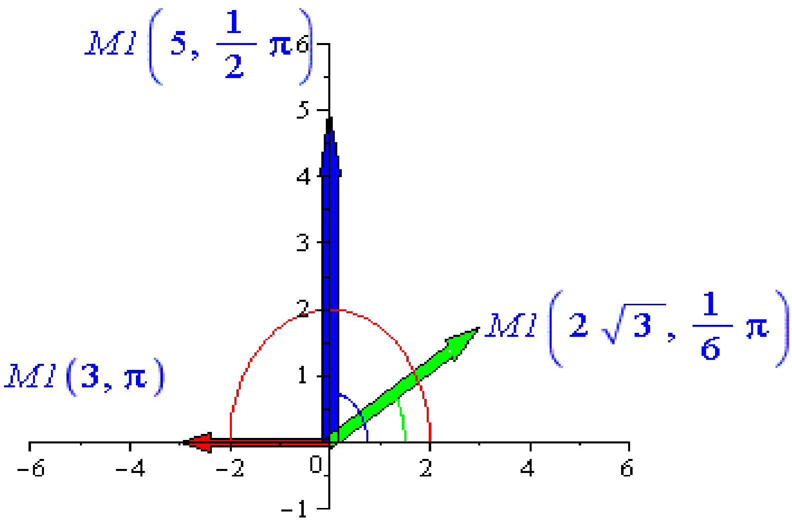
```
L3a:= arc([0,0],2,0..arctan(b3/a3)+1*Pi,color=red):
plots[display](L1,L2,L3,L1a,L2a,L3a, axes=normal,
view=[-3..4,-1..6]);
```
(4 -rasim)

Kompleks sonlarning nuqtalarini qirish.

```
> with(plots): Nuqta1:=textplot([[r1*cos(phi1),
r1*sin(phi1), 'M1(r1,phi1)']],color=blue,align=Right,
font=[TIMES,ROMAN,14]):
> Nuqta2:=textplot([[r2*cos(phi2),r2*sin(phi2),
'M1(r2,phi2)']], color=blue,align=Left, font=[TIMES,ROMAN,14]):
> Nuqta3:=textplot([[r3*cos(phi3),r3*sin(phi3)+1,
'M1(r3,phi3)']], color=blue,align=Left, font=[TIMES,ROMAN,14]):
> KST:= plots[display](KS,Nuqta1,Nuqta2,Nuqta3, axes=normal,view=[-8..8,-3..8],
scaling=constrained);
```

$$KST := PLOT(\ldots)$$

```
> display([KST]);
```
(4 –rasm)



4 -rasm.

Xullosa. Maple dasturining imkoniyatlarini kompleks sinlar uchun qo‘llanishi o‘quvchida tasviriy fikirlash, masalani yechishning programmalash va animatsiyalash imkonoyatini hamda kompleks sonlarni koeffitsentlariga qarab tez va aniq qurish va qo‘shishda Maple dasturini qo‘llash usullari ko‘rsatilgan.

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