

KOMPLEKS SONLAR USTIDA CHIZIQLI AMALLAR MAPLE TIZIMIDA

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Annotatsiya:

Kompleks son haqida asosiy tushunchalar va ta’riflar berilgan. Algebraik shakldagi kompleks sonlar ustida qo’shish va ayirish amalalari Maple tizimida bajarilgan.

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KIRISH. Hozirgi vaqtida har qanday jiddiy hisob-kitoblar, qoida tariqasida, kompyuterlarda va birinchi navbatda, shaxsiy kompyuterlarda amalga oshiriladi. Ushbu maqolada Maple dasturidan foydalanib kompleks sonlar uchun tuzilgan matematik modellarning samarali va undan foydalanib tahlil va qaror qabul qilishda axamiyatli ekanligini ko’rsatamiz.

1. Asosiy ta’riflar.

1-ta’rif. *a va b –haqiqiy sonlar uchun yozilgan $z = a + ib$ ko‘rinishidagi ifodaga kompleks son deb aytildi.*

Bunda $i = \sqrt{-1}$ ($i^2 = -1$) tenglik bilan aniqlanuvchi mavhum birlik deb ataluvchi birlik. z kompleks sonning haqiqiy va mavhum qismlari quyidagicha belgilanadi:

$$\operatorname{Re} z = a, \quad \operatorname{Im} z = b.$$

Xususiy holda, agar $a = 0$ bo'lsa, u holda $z = 0 + ib$ sonni *sof mavhum* son, agar $b = 0$ bo'lsa, u holda $z = a + i \cdot 0 = a$, ya'ni haqiqiy son hosil bo'ladi. Shunday qilib, haqiqiy va mavhum sonlar z kompleks sonlarning xususiy hollaridir.

Kompleks sonning $z = a + bi$ ko'rinishdagi yozuvi uning *algebraik shakli* deyiladi.

2-ta'rif. Agar ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonlarning haqiqiy qismlari va mavhum qismlari o'zaro teng bo'lsa, bu kompleks sonlar *teng*, ya'ni $z_1 = z_2$ bo'ladi, ($\operatorname{Re} z_1 = \operatorname{Re} z_2$ va $\operatorname{Im} z_1 = \operatorname{Im} z_2$ bo'lsa, $z_1 = z_2$ hisoblanadi).

3-ta'rif. $z = a + ib$ kompleks sonning haqiqiy va mavhum qismi nolga teng bo'lsagina, u nolga teng bo'ladi, ya'ni agar $a = 0$ va $b = 0$ bo'lsagina $z = 0$, va aksincha.

4-ta'rif. $z = a + ib$ va $\bar{z} = a - ib$ kompleks sonlar *qo'shma kompleks sonlar* deyiladi.

5-ta'rif. $z_1 = a + ib$ va $z_2 = -a - ib$ kompleks sonlar *qarama-qarshi kompleks sonlar* deyiladi.

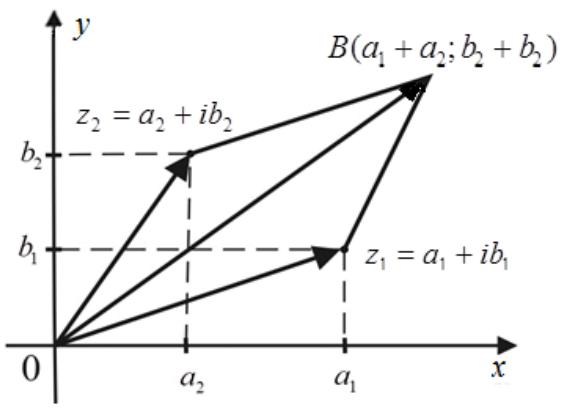
2. Algebraik shakildagi kompleks sonlar ustida amallar

Kompleks sonlarni qo'shish

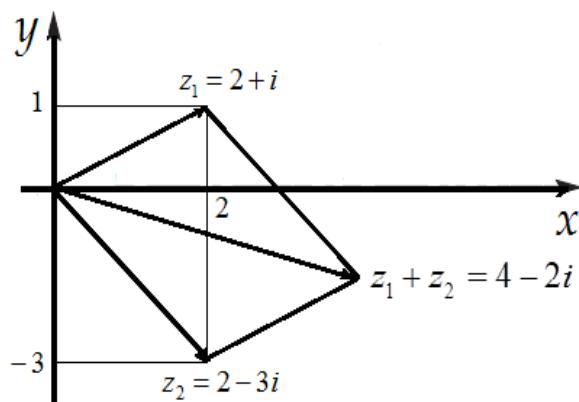
Ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonning *yig'indisi* deb,

$$z_1 + z_2 = (a_1 + ib_1) + (a_2 + ib_2) = (a_1 + a_2) + i(b_1 + b_2)$$

tenglik bilan aniqlanuvchi kompleks songa aytildi. Bu formuladan vektorlar bilan ifodalangan kompleks sonlarni qo'shish vektorlarni qo'shish qoidasi bo'yicha bajarilishi kelib chiqadi (1-rasm).



1-rasm.



1a -rasm.

1-misol. Ushbu $z_1 = 2 + i$ va $z_2 = 2 - 3i$ kompleks sonlarning *yig'indisini* toping.

$$\text{Yechish. } z_1 + z_2 = (2 + i) + (2 - 3i) = (2 + 2) + i(1 - 3) = 4 - 2i.$$

(1a-rasm)

Maple dasaturi

> restart;
> $x1:=2:y1:=1: x2:=-2:y2:=-3:$

$z_1 = 2 + i$ kompleks sonni aniqlash:

> $z1:=x1+y1*I;$ $z1 := 2 + I$
> $y1:=\text{Im}(z1); x1:=\text{Re}(z1);$ $y1 := 1$ $x1 := 2$
> polar(z1); $\text{polar}\left(\sqrt{5}, \arctan\left(\frac{1}{2}\right)\right)$

$z_2 = 2 - 3i$ kompleks sonni aniqlash:

> $z2:=x2+y2*I;$ $z2 := 2 - 3I$
> $y2:=\text{Im}(z2); x2:=\text{Re}(z2);$ $y2 := -3$ $x2 := 2$
> polar(z2); $\text{polar}\left(\sqrt{13}, -\arctan\left(\frac{3}{2}\right)\right)$

$z_3 = z_1 + z_2 = 4 - 2i$ kompleks sonni aniqlash:

> $z3:=z1+z2;$ $z3 := 4 - 2I$
> $y3:=\text{Im}(z3); x3:=\text{Re}(z3);$ $y3 := -2$ $x3 := 4$
> polar(z3); $\text{polar}\left(2\sqrt{5}, -\arctan\left(\frac{1}{2}\right)\right)$

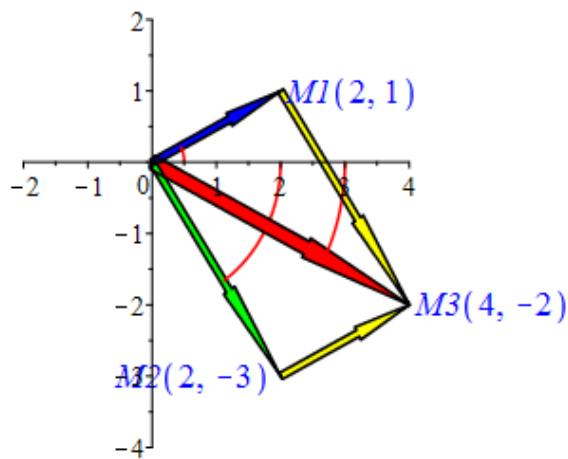
Kompleks sonlarni vektorlarini qurish:

> with(plottools):
Vz1:=arrow([0,0], [x1,y1],.1,.2,.4, color=blue):
Vz1a:=arc([0,0],.5,0..arctan(Im(z1),Re(z1)),
color=red):
Vz2:=arrow([0,0], [x2,y2],.1,.2,.4, color=green):
Vz2a:=arc([0,0],2,0..arctan(Im(z2),Re(z2)),
color=red):
Vz3:=arrow([0,0], [x3,y3],.2,.3,.4, color=red):
Vz3a:=arc([0,0],3,0..arctan(Im(z3),Re(z3)),
color=red):
Vz13:=arrow([x1,y1],[x3,y3],.1,.2,.4,color=yellow):
Vz23:=arrow([x2,y2],[x3,y3],.1,.2,.4,color=yellow):
plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a,Vz13,
Vz23, axes=normal,view=[-4..4,-4..4], scaling=constrained);

Kompleks sonlarning nuqtalarini qirish.

> with(plots): Nuqta1:=textplot([[x1,y1,'M1(x1,y1)']], color=blue, align=Right,
font=[TIMES,ROMAN,14]):
> Nuqta2:=textplot([[x2,y2,'M2(x2,y2)']], color=blue, align=Left,
font=[TIMES,ROMAN,14]):
> Nuqta3:=textplot([[x3,y3,'M3(x3,y3)']], color=blue, align=Right,
font=[TIMES,ROMAN,14]):
> KSA:=plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a, Vz13,
Vz23,Nuqta1,Nuqta2,Nuqta3,axes=normal,

view=[-2..4,-4..2],scaling=constrained); (1a-rasm)



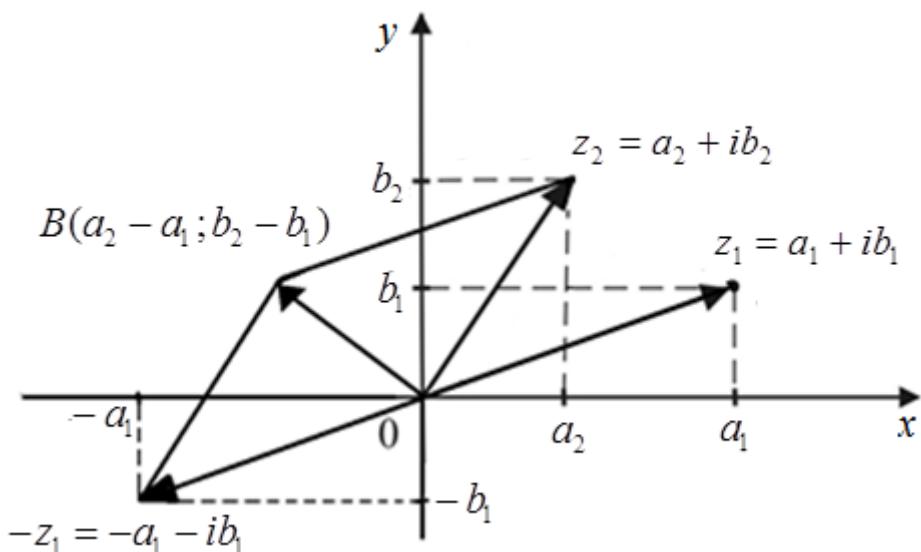
1a-rasm.

Kompleks sonlarni ayirish.

Ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonning ayirmasi deb, shunday songa aytiladiki, u z_2 ga qo'shilganda yig'indida z_1 kompleks son hosil bo'ladi (2-rasm). Demak,

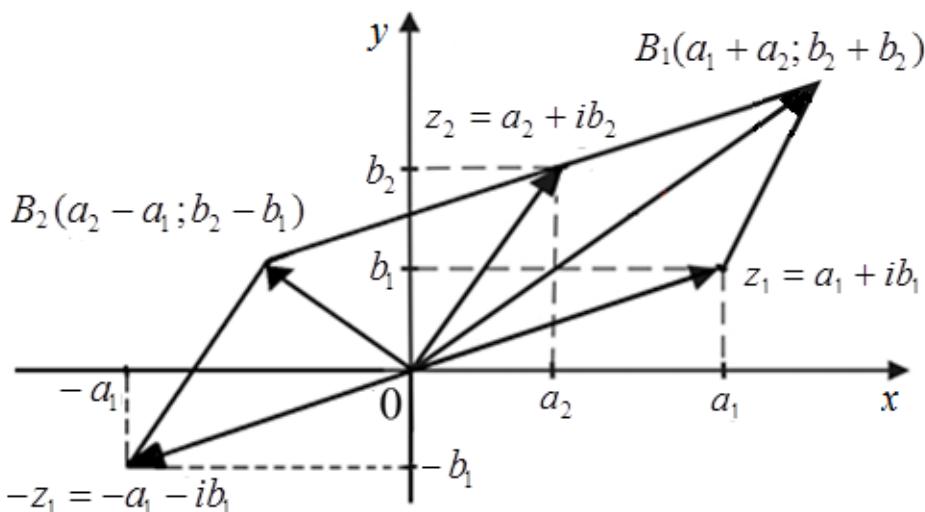
$$z_2 - z_1 = (a_2 + ib_2) - (a_1 + ib_1) = (a_2 - a_1) + i(b_2 - b_1).$$

Demak, grafigini qurishda $z_2 - z_1$ ayirma $z_2 + (-z_1)$ yig'indi bo'lishini e'tiborga olamiz.



2-rasm.

Ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonning yig'indisi va ayirmasini bitta koordinatalar sistemasida quyidagicha quramiz (3-rasm).



3-rasm.

Shuni ta'kidlab o'tamiz, ikki kompleks son ayirmasining moduli kompleks tekislikda shu sonlarni ifodalovchi nuqtalar orasidagi masofaga teng:

$$|z_2 - z_1| = \sqrt{(a_2 - a_1)^2 + (b_2 - b_1)^2}.$$

2-misol. Ushbu $z_1 = 4 - 3i$ va $z_2 = 2 + 6i$ kompleks sonlarning yig'indis va ayirmasini toping.

Yechish.

$$z_2 + z_1 = (2 + 6i) + (4 - 3i) = (2 + 4) + i(6 - 3) = 6 + 3i.$$

$$z_2 - z_1 = (2 + 6i) - (4 - 3i) = (2 - 4) + i(6 + 3) = -2 + 9i.$$

Bu kompleks sonlarni ayirmasini Maple dasturida qurish ucun

$$z_2 - z_1 = z_2 + (-z_1)$$

qoidani e'tiborga olamiz(2-rasm).

Maple dasturi

> restart;

$z_1 = 4 - 3i$ kompleks sonni aniqlash:

> x1:=4:y1:=-3: x2:=2:y2:=6:

> z1:=x1+y1*I; $z1 := 4 - 3i$

> x1:=Re(z1); y1:=Im(z1); $x1 := 4$ $y1 := -3$

> polar(z1); $\text{polar}\left(5, -\arctan\left(\frac{3}{4}\right)\right)$

$z_2 = 2 + 6i$ kompleks sonni aniqlash:

> x2:=2:y2:=6:I; $z2 := 2 + 6i$

> x2:=Re(z2); y2:=Im(z2); $x2 := 2$ $y2 := 6$

> polar(z2); $\text{polar}\left(2\sqrt{10}, \arctan(3)\right)$

$z_3 = z_2 + z_1 = 6 + 3i$ kompleks sonni aniqlash:

```
> z3:=z2+z1;           z3 := 6 + 3 I
> y3:=Im(z3);x3:=Re(z3);   y3 := 3  x3 := 6
> polar(z3);          polar(3 √5, arctan(1/2))
```

$z_4 = z_2 - z_1 = -2 + 9i$ kompleks sonni aniqlash:

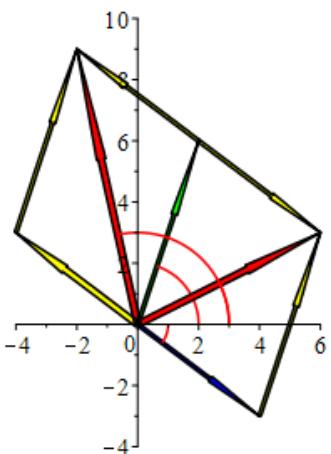
```
> z4:=z2-z1;           z4 := -2 + 9 I
> y4:=Im(z4);x4:=Re(z4);   y4 := 9  x4 := -2
> polar(z4);          polar(√85, -arctan(9/2) + π)
```

Kompleks sonlarning yig‘indis va ayirmasini qurish:

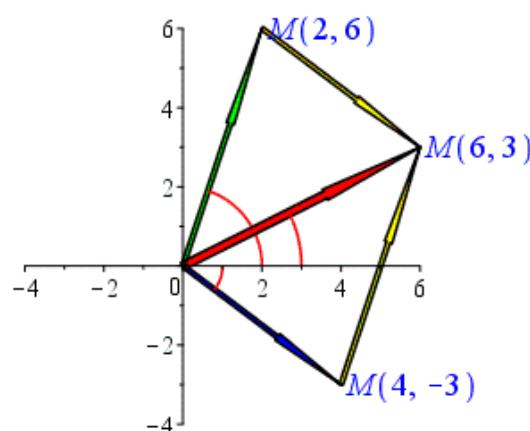
```
> with(plottools):
Vz1:=arrow([0,0], [x1,y1], .1, .2, .4, color=blue);
Vz1a:=arc([0,0],1,0..arctan(Im(z1),Re(z1)), color=red):
Vz2:=arrow([0,0],[x2,y2], .1, .2,.4,color=green):
Vz2a:=arc([0,0],2,0..arctan(Im(z2),Re(z2)),
color=red):
Vz3:=arrow([0,0], [x3,y3], .2, .3, .4, color=red):
Vz3a:=arc([0,0],3,0..arctan(Im(z3),Re(z3)),
color=red):
Vz13:=arrow([x1,y1], [x3,y3], .1, .2, .4, color=yellow):
Vz23:=arrow([x2,y2], [x3,y3], .1, .2, .4, color=yellow):
Vz4:=arrow([0,0], [x4,y4], .2, .3, .4,color=red):
Vz4a:=arc([0,0],3,0..arctan(Im(z4),Re(z4)),
color=red):
Vz1q:=arrow([0,0], [-x1,-y1], .2, .3, .4, color=yellow):
Vz14:=arrow([-x1,-y1], [x4,y4], .1, .2, .4, color=yellow):
Vz24:=arrow([x2,y2], [x4,y4], .1, .2, .4, color=yellow):
plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a,Vz13,
Vz23,Vz4,Vz4a,Vz14,Vz24,Vz1q, axes=normal,
view=[-4..6,-4..10],scaling=constrained);(4-rasm)
```

Kompleks sonlarning yig‘indisini koordinatalari bilan qurish:

```
> with(plots): Nuqta1:=textplot([[x1,y1,’M(x1,y1)’]], color=blue,align=Right,
font=[TIMES,ROMAN,14]):
> Nuqta2:=textplot([[x2,y2,’M(x2,y2)’]], color=blue,align=Right,
font=[TIMES,ROMAN,14]):
> Nuqta3:=textplot([[x3,y3,’M(x3,y3)’]], color=blue,align=Right,
font=[TIMES,ROMAN,14]):
> KSA:=plots[display](Vz1,Vz1a,Vz2,Vz2a,Vz3,Vz3a,
Vz13, Vz23,Nuqta1,Nuqta2,Nuqta3, axes=normal, view=[-4..6,-4..6],scaling=constrained);
(5-rasm)
```



4-rasm.



5-rasm.

Xullosa. Maple dasturining imkoniyatlarini kompleks sinlar uchun qo'llanishi o'quvchida tasviriy fikirlash, masalani yechishning programmalash va animatsiyalash imkonoyatini hamda kompleks sonlarni koeffitsientlariga qarab tez va aniq qurish va qo'shishda Maple dasturini qo'llash usullari ko'rsatilgan.

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