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GEOMETRIC REPRESENTATION OF COMPLEX NUMBERS IN THE MAPLE SYSTEM

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A R T I C L E I N F O.

Keywords: Complex number, real number, abstract unit, algebraic form, abstract number, joint complex number, opposite complex number, modulus, argument, trigonometric form.

Abstract

The article describes the mathematical apparatus of complex numbers: the basic concepts and definitions are given, the solutions of problems on the definition and geometric interpretation of complex numbers in algebraic and trigonometric forms in the Maple system are performed.[1,2,3,7,8,9].

KIRISH: Hozirgi vaqtda har qanday jiddiy hisob-kitoblar, qoida tariqasida, kompyuterlarda va birinchi navbatda, shaxsiy kompyuterlarda amalga oshiriladi. Ushbu maqolada Maple dasturidan foydalanib kompleks sonlar uchun tuzilgan matematik modellarning sifati va undan raqamli usullarda foydalanib tahlil va qaror qabul qilishda axamiyatli ekanligi ko‘rsatilgan

1.Asosiy ta’riflar.

1-ta’rif. *a* va *b –haqiqiy sonlar uchun yozilgan* $z = a + ib$ ko‘rinishidagi ifodaga *kompleks son* deb aytiladi.

Bunda $i = \sqrt{-1}$ ($i^2 = -1$) tenglik bilan aniqlanuvchi *mavhum birlik* deb ataluvchi birlik. *z* kompleks sonning haqiqiy va mavhum qismlari quyidagicha belgilanadi:

$$\operatorname{Re} z = a, \quad \operatorname{Im} z = b.$$

Xususiy holda, agar $a = 0$ bo‘lsa, u holda $z = 0 + i\,b$ sonni *sof mavhum* son, agar $b = 0$ bo‘lsa, u holda $z = a + i \cdot 0 = a,$ ya’ni haqiqiy son hosil bo‘ladi. Shunday qilib, haqiqiy va mavhum sonlar z kompleks sonlarning xususiy hollaridir.

Kompleks sonning $z = a + bi$ ko‘rinishdagi yozuvi uning *algebraik shakli* deyiladi.

2-ta’rif. Agar ikkita $z_1 = a_1 + ib_1$ va $z_2 = a_2 + ib_2$ kompleks sonlarning haqiqiy qismlari va mavhum qismlari o‘zaro teng bo‘lsa, bu kompleks sonlar *teng*, ya’ni $z_1 = z_2$ bo‘ladi, ($\operatorname{Re} z_1 = \operatorname{Re} z_2$ va $\operatorname{Im} z_1 = \operatorname{Im} z_2$ bo‘lsa, $z_1 = z_2$ hisoblanadi).

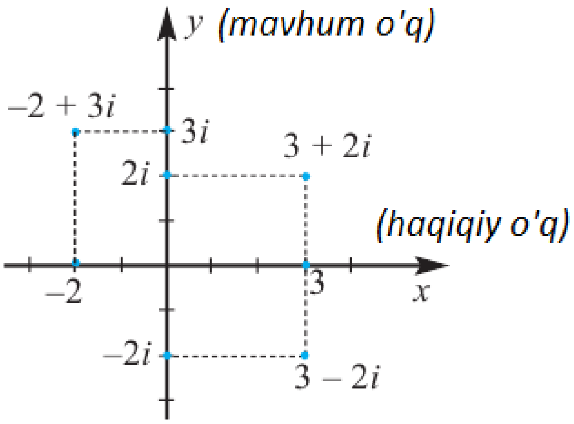
3-ta'rif. $z = a + ib$ kompleks sonning haqiqiy va mavhum qismi nolga teng bo'lsagina, u nolga teng bo'ladi, ya'ni agar $a = 0$ va $b = 0$ bo'lsagina $z = 0$, va aksincha.

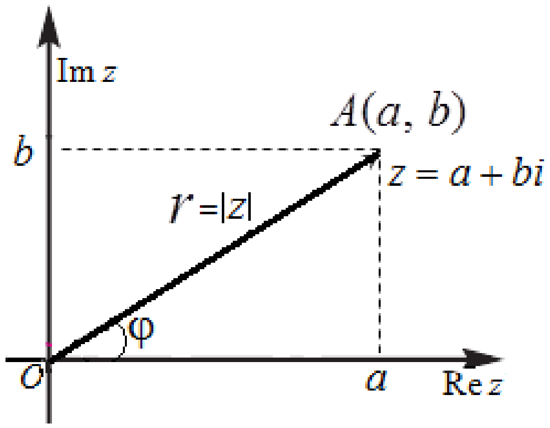
4-ta'rif. $z = a + ib$ va $\overline{z} = a - ib$ kompleks sonlar *qo'shma kompleks sonlar* deyiladi.

5-ta'rif. $z_1 = a + ib$ va $z_2 = -a - ib$ kompleks sonlar *qarama-qarshi kompleks sonlar* deyiladi.

2. Kompleks sonlarning geometrik talqini va trigonometrik shakli. Har qanday $z = a + ib$ kompleks soni uchun Oxy tekislikda *a* va *b* koordinatali $A(a;b)$ nuqta mos keladi. Bu nuqta kompleks sonning *affeksi* diyiladi. Tekislikning har bir nuqtasiga kompleks son mos keladi va aksincha. Kompleks tekislikda z sonni tasvirlovchi nuqtani z nuqta deb ataymiz (1-rasm). Ox o'qda yotuvchi nuqtalarga haqiqiy sonlar mos keladi, Oy o'qda yotuvchi nuqtalar sof mavhum sonlarni tasvirlaydi. Shu sababli Ox o'q *haqiqiy* o'q. Oy o'q *mavhum* o'q deyiladi.

1-musol. $z_1 = -2 + 3i$, $z_2 = 3 + 2i$, $z_3 = 3 - 2i$ kompleks sonlarning nuqtasini tasvirini quyidagi 1-rasmda ko'ramiz.





1-rasm. 2-rasm.

Koordinatalar boshini qutb boshi nuqtasi deb, Ox o'qning musbat yo'nalishini qutb o'qi deb kompleks tekislikda koordinatalarning qutb sistemasini kiritamiz. φ va r larni $A(a;b)$ nuqtaning qutb koordinatalari deymiz A nuqtaning qutb radiusi r, ya'ni A nuqtadan qutbgacha, O nuqtagacha bo'lgan masofa z kompleks sonning *moduli* deyiladi va *r* yoki $|z|$ kabi belgilanadi(2-rasim).

$$r = |z| = \sqrt{a^2 + b^2} \quad (1)$$

A nuqtaning qutb burchagi φ ni z kompleks sonning *argumenti* quyidagicha belgilanadi:

$$Argz = arctg\,\frac{b}{a}$$

Ushbu $a = r\cos\varphi,\ b = r\sin\varphi$ tengliklarni hisobga olib, z kompleks sonni trigonometrik ko'rinishda ifodalash mumkin:

$$z = a + bi = r(\cos\varphi + i\sin\varphi), \quad (2)$$

bunda $r = |z| = \sqrt{a^2 + b^2}$, $tg\,\varphi = \dfrac{b}{a}$.

$$\varphi = \arg z = \begin{cases} arctg\dfrac{b}{a}, \text{ агар } a > 0,\ b \geq 0 \text{ бўлса}, \\ \pi - arctg\dfrac{b}{a} +, \text{ агар } a < 0 \text{ бўлса}, \\ arctg\dfrac{b}{a} + 2\pi, \text{ агар } a > 0,\ b < 0 \text{ бўлса}. \end{cases} \qquad (2')$$

Yozuvning bu shakli

$$z = r(\cos\varphi + i\sin\varphi) \quad (3)$$

kompleks sonning *trigonometrik shakli* deyiladi.

2-misol. $z = -1 + \sqrt{3}i$ kompleka sonni trigonometrik shaki aniqlang va geometrik tasvirini quring

Yechish: Kompleks sonning *moduli va argumenti* quyidagicha:

$$R = \sqrt{a^2 + b^2} = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

$$\mathrm{tg}\,\varphi = \frac{b}{a} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$$

Bunda φ burchak koordinatalar sistemasining II choragiga tegishli

(3-rasm):

$$\varphi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Demak, kompleka sonni trigonometrik shaki va tasviri-grafigi

(3a-rasim):

$$z = -1 + \sqrt{3}i = 2(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$$

M a p l e d a s t u r i

> restart; a:=-1; b:=sqrt(3); $a := -1 \quad b := \sqrt{3}$

> z:= a+b*I; $z := -1 + \mathrm{I}\sqrt{3}$

> polar(z); r:=abs(z); phi:=argument(z);

$\mathrm{polar}\left(2, \frac{2\pi}{3}\right) \quad r := 2 \quad \phi := \frac{2\pi}{3}$

> z:=abs(z)*(Cos(phi)+I*Sin(phi));

$z := 2\,Cos\left(\frac{2\pi}{3}\right) + 2\,\mathrm{I}\,Sin\left(\frac{2\pi}{3}\right)$

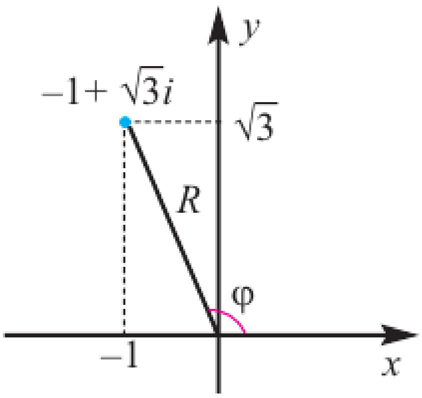
> with(plottools):

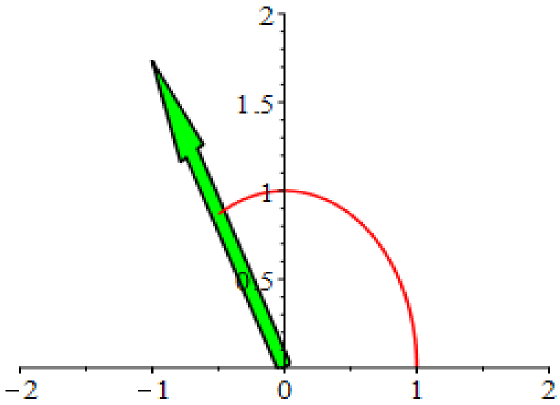
L1:=arrow([0,0], [a,b],.1,.2,.3, color=green):

L2:=arc([0,0],1,0..phi,color=red):

plots[display](L1,L2, axes=normal,view=[-r..r,

-r..r],scaling=constrained); (3a-rasm)





3-rasm. 3a-rasm.

$z = -1 + \sqrt{3}i$ kompleka sonni algebraik va trigonometrik shaki bo‘yicha qurish dasturini tuzamiz:

M a p l e d a s t u r i

> restart;

> a:=-1: b:=sqrt(3):

> z:=a+b*I; a:=Re(z); $z := -1 + \mathrm{I}\sqrt{3}$

> b:=Im(z); $a := -1 \quad b := \sqrt{3}$

> r:=abs(z); phi:=argument(z); $r := 2 \quad \phi := \frac{2\pi}{3}$

> z1:=abs(z)*(Cos(phi)+I*Sin(phi));

$z1 := 2\,Cos\left(\frac{2\pi}{3}\right) + 2\,\mathrm{I}\,Sin\left(\frac{2\pi}{3}\right)$

> with(plottools): Vz:=arrow([0,0], [a,b], .1, .2, .4, color=blue):

> Vza:=arc([0,0],.5,0..arctan(b,a),color=red):

> with (plots): Nuqta: = textplot ([[a,b,'M(a,b)']], color = blue, align = Left, font= [TIMES,ROMAN,14]):

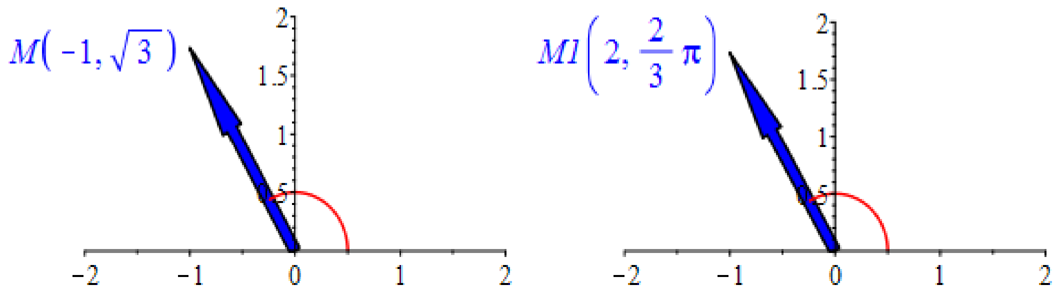
> KSA: = plots[display](Vz,Vza,Nuqta, axes=normal,

View = [-r..r,0..r], scaling=constrained);(3b-rasm)

> with(plots): Nuqta:=textplot([[r*cos(phi),r*sin(phi),'M1(r,phi)']], color=blue,align=Left, font=[TIMES,ROMAN,14]):

> KST:= plots[display](Vz,Vza, Nuqta, axes=normal,

view=[-r..r,0..r],scaling=constrained); (3c-rasm)



3b -rasm. 3c -rasm.

3-misol. Quyidagi kompleks sonlarni trigonometrik shaklda tasvirlang:

$$1)\ z=3+\sqrt{3}\,i;\ 2)\ z=5i;\ 3)\ z=-3.$$

Yechish. 1) z kompleks sonning modulini va argumentining qiymatini topamiz. Bu erda $a=3,\ \ b=\sqrt{3}$ ga teng, (1) formulaga ko‘ra topamiz:

$$r=\sqrt{3^2+\left(\sqrt{3}\right)^2}=\sqrt{9+3}=\sqrt{12}=\sqrt{4\cdot 3}=2\sqrt{3}.$$

Argumentini (2’) formuladan $tg\,\varphi=\dfrac{b}{a}=\dfrac{\sqrt{3}}{3}$ bo‘lgani uchun, $z=3+\sqrt{3}\,i$ nuqta I chorakda yotgani uchun $\varphi=\mathrm{arc}\,tg\dfrac{\sqrt{3}}{3}=\dfrac{\pi}{6}$ ni Hosil qilamiz. Bunda (3) formuladan foydalanib, $z=3+\sqrt{3}\,i$ sonning trigonometrik shaklini Hosil qilamiz:

$$z=3+\sqrt{3}\,i=2\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right).$$

2) $z=5i$ kompleks sonning modulini va argumentining qiymatini topamiz. Bu yerda $a=0,\ \ b=5$ ga teng, (1) formulaga ko‘ra topamiz: $r=\sqrt{0^2+(5)^2}=\sqrt{0+25}=5.$

Argumentini (2) formuladan $tg\varphi = \frac{b}{a} = \frac{5}{0} = \infty$ bo'lgani uchun, $z = 5i$ nuqta I chorak va II chorak o'rtasida yotgani uchun $\varphi = \mathrm{arc}\,tg(\frac{5}{0}) = arctg(\infty) = \frac{\pi}{2}$ ni hosil qilamiz. Bunda (3) formuladan foydalanib, $z = 5i$ sonning trigonometrik shaklini hosil qilamiz: $z = 5i = 5\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$.

3) $z = -3$ kompleks sonning modulini va argumentining qiymatini topamiz. Bu erda $a = -3,\ \ b = 0$ ga teng, (1) formulaga ko'ra topamiz: $r = \sqrt{(-3)^2 + (0)^2} = \sqrt{9+0} = 3$. Argumentini (2) formuladan $tg\varphi = \frac{b}{a} = \frac{0}{-3} = 0$ bo'lgani uchun, $z = -3$ nuqta II chorak va III chorak o'rtasida yotgani uchun $\varphi = \mathrm{arc}\,tg(\frac{0}{-3}) = arctg(0) = \pi$ ni hosil qilamiz. Bunda (3) formuladan foydalanib, $z = -3$ sonning trigonometrik shaklini hosil qilamiz: $z = -3 = 3(\cos\pi + i\sin\pi)$.

Misoldagi 1) $z = 3 + \sqrt{3}i$; 2) $z = 5i$; 3) $z = -3$ kompleks sonlarni trigonometrik shaklni aniqlash va geometrik tasvirlash.

M a p l e d a s t u r i

> restart;

> a1:=3: b1:=sqrt(3): z1:=a1+b1*I; $z1 := 3 + \mathrm{I}\sqrt{3}$

> polar(z1); r1:=abs(z1); phi1:=argument(z1);

$\mathrm{polar}\left(2\sqrt{3}, \frac{\pi}{6}\right) \quad r1 := 2\sqrt{3} \quad \phi 1 := \frac{\pi}{6}$

> z1:=r1*(Cos(phi1)+I*Sin(phi1));

$z1 := 2\sqrt{3}\left(Cos\left(\frac{\pi}{6}\right) + \mathrm{I}\,Sin\left(\frac{\pi}{6}\right)\right)$

> a2:=0: b2:=5: z2:=a2+b2*I; $z2 := 5\,\mathrm{I}$

> polar(z2); r2:=abs(z2); phi2:=argument(z2);

$\mathrm{polar}\left(5, \frac{\pi}{2}\right) \quad r2 := 5 \quad \phi 2 := \frac{\pi}{2}$

> z2:=r2*(Cos(phi2)+I*Sin(phi2)); #z2:=evalc(%);

$z2 := 5\,Cos\left(\frac{\pi}{2}\right) + 5\,\mathrm{I}\,Sin\left(\frac{\pi}{2}\right)$

> a3:=-3: b3:=0: z3:=a3+b3*I; $z3 := -3$

> polar(z3); r3:=abs(z3); phi3:=argument(z3);

$\mathrm{polar}(3, \pi) \quad r3 := 3 \quad \phi 3 := \pi$

> **z3:=r3*(Cos(phi3)+I*Sin(phi3));** $z3 := 3\,Cos(\pi) + 3\,I\,Sin(\pi)$

Kompleks sonlarni vektorlarini qurish:

> with(plottools):

L1:= arrow([0,0], [a1,b1], .2, .4, .2, color=green):

L1a:= arc([0,0],1.5,0..arctan(b1/a1),color=green):

L2:= arrow([0,0], [a2,b2], .3, .4, .2, color=blue):

L2a:= arc([0,0],.75,0..1*Pi/2,color=blue):

L3:= arrow([0,0], [a3,b3], .1, .3, .2, color=red):

L3a:= arc([0,0],2,0..arctan(b3/a3)+1*Pi,color=red):

plots[display](L1,L2,L3,L1a,L2a,L3a, axes=normal,

view=[-3..4,-1..6]); (4 -rasim)

Kompleks sonlarning nuqtalarini qirish.

> with(plots): Nuqta1:=textplot([[r1*cos(phi1),

r1*sin(phi1), 'M1(r1,phi1)']],color=blue,align=Right,

font=[TIMES,ROMAN,14]):

> Nuqta2:=textplot([[r2*cos(phi2),r2*sin(phi2),

'M1(r2,phi2)']], color=blue,align=Left, font=[TIMES,ROMAN,14]):

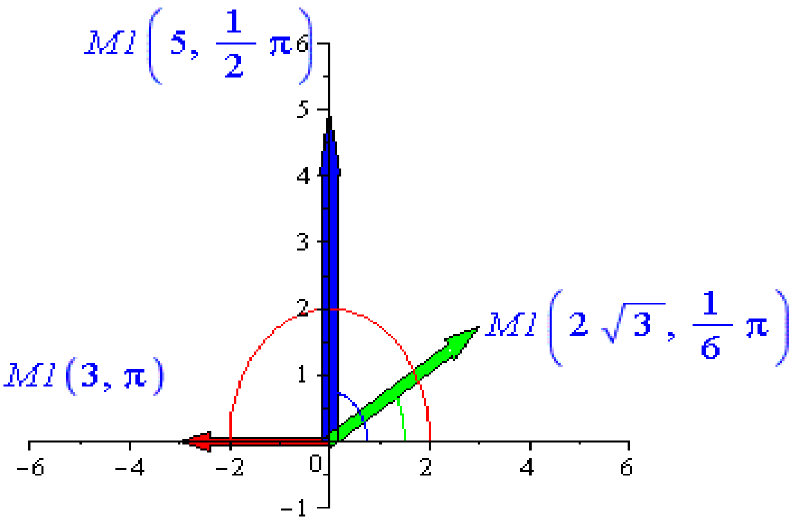
> Nuqta3:=textplot([[r3*cos(phi3),r3*sin(phi3)+1,

'M1(r3,phi3)']], color=blue,align=Left, font=[TIMES,ROMAN,14]):

> KST:= plots[display](KS,Nuqta1,Nuqta2,Nuqta3, axes=normal,view=[-8..8,-3..8], scaling=constrained);

KST := PLOT(...)

> display([KST]); (4 –rasm)



4 -rasm.

Xullosa. Maple dasturining imkoniyatlarini kompleks sinlar uchun qo‘llanishi o‘quvchida tasviriy fikirlash, masalani yechishning programmalash va animatsiyalash imkonoyatini hamda kompleks sonlarni koeffitsentlariga qarab tez va aniq qurish va qo‘shishda Maple dasturini qo‘llash usullari ko‘rsatilgan.

Adabiyotlar

1. Матросов А.В. (2001). Maple 6. Решение задач высшей математики и механики. СПб. БХВ-Петербург.
2. Mirzakarimov E.M. (2014, 2015). Maple dasturi yordamida Oliy matematika masalalarini yechish. 1, 2,3q, T. Adabiyot uchqunlari.
3. Mirzakarimov, E.M., (2021).Maple tizimida to‘g‘ri to‘rtburchakli membrananing erkin tebranishini aniqlash. *Scientific Bulletin of Namangan State University* 1(8), 3-9.
4. Mirzakarimov, E.M., (2022).To‘g‘riburchakli membrananing erkin tebranishini boshlang‘ich shartlar bo‘yicha aniqlashda maple tizimidan foydalanish. *Scientific Bulletin of Namangan State University* 1(1), 53-61.
5. Mirzakarimov, E.M., & Fayzullaev, J.S. (2020). Improving the quality and efficiency of teaching by developing students* mathematical competence using the animation method of adding vectors to the plane using the maple system. *Scientific Bulletin of Namangan State University*, 2(9), 336-342.
6. Mirzakarimov, E.M., & Faizullaev, J.I. (2019). Method of teaching the integration of information and educational technologies in a heterogeneous parabolic equation. *Scientific Bulletin of Namangan State University*, 1(5), 13-17.
7. Mirzaboevich, M.E., & Ugli, E. M.I. (2021). Using Maple Programs in Higher Mathematics Triangle Problem Constructed on Vectors in Space. *Central Asian journal of mathematical theory and computer sciences*, 2(11), 44-50. Retrieved from http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/123
8. Мирзабоевич,М.Э., &Исмоилджонович, Ф.Д. (2021). Выполнять Линейные Операции Над Векторами В Пространстве В Системе Maple. *Central asian journal of mathematical theory and computer ciences*, 2(12), 10-16. http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/137
9. Мирзабоевич,М.Э.,(2022). Использовать Систему Maple Для Определения Свободных Колебаний Прямоугольной Мембраны При Начальных Условиях. *Central Asian Journal Of Mathematical Theory And Computer Sciences*, *3*(1),9-18. https://cajmtcs.centralasianstudies.org/index.php/ CAJMTCS/ article/view/148
10. Fayzullayev, J. I. (2020). Mathematical competence development method for students through solving the vibration problem with a maple system. *Scientific Bulletin of Namangan State University*, *2*(8), 353-358.
11. Mirzaboevich, M. E. (2021). Using Maple Programs in Higher Mathematics. Triangle Problem Constructed on Vectors in Space. *Central Asian journal of mathematical theory and computer sciences*, *2*(11), 44-50.
12. Fayzullayev, J. I. (2021). Fundamental fanlar yordamida texnika oliy ta’lim muassasalari talabalarining kasbiy kompetentligini rivojlantirish. *Oriental renaissance: Innovative, educational, natural and social sciences*, *1*(10), 454-461.

13. Мирзабоевич, М. Э. (2022). Создание корреляционной таблицы в системе maple. *Central asian journal of mathematical theory and computer sciences*, *3*(10),1-7. https://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/236
14. Mirzakarimov, E., & Yo‘ldashaliyev , A. (2023). Tekislikda Vektorlar Ustida Qo‘Shish Amallarini Maple Tizimida Bajarish. *Research and Implementation*. извлечено от https://fer-teach.uz/index.php/rai/article/view/834