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FORMULATION AND EXPRESSION OF A LINEAR PROGRAMMING PROBLEM IN VARIOUS FORMS

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Abstract

The formulation of the linear programming problem. The transformation of a linear inequality with n unknowns into a linear equation with n + 1 unknowns. Equivalent substitutions in a linear programming problem. The canonical form and expression in various forms of a linear programming problem.

In general, the linear programming problem is expressed as follows:

Given a linear function

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n \qquad (1.1)$$

and linear constraints

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le (\ge)\, b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le (\ge)\, b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le (\ge)\, b_m \end{cases}, \qquad (1.2)$$

$$x_1 \ge 0,\ x_2 \ge 0, \ldots,\ x_n \ge 0. \qquad (1.3)$$

Find such values of unknowns satisfying conditions (1.2) and (1.3) that deliver the maximum (minimum) value to the linear function (1.1).

The conditions (1.2) and (1.3) of the problem are called its boundary conditions, and the linear function (1.1) is the goal or objective function of the problem. Since all boundary conditions and the objective function of this problem are linear, problem (1.1) – (1.3) is called a linear programming problem.

In specific problems, condition (1.2) can be a system of equations, a system of inequalities of the form «≤» or «≥», or a mixed system. However, it can be shown that a problem of the form (1.1) – (1.3) can be reduced to the form

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n, \quad (1.4)$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n = b_m \end{cases}, \quad (1.5)$$

$$x_1 \geq 0,\ x_2 \geq 0, \ldots, x_n \geq 0. \quad (1.6)$$

Indeed, consider a linear inequality with n unknowns

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n \leq b. \quad (1.7)$$

To convert this inequality into an equation, add an unknown number to its smaller side $x_{n+1} \geq 0$. As a result, we obtain a linear equation with $n + 1$ unknown

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n + x_{n+1} = b. \quad (1.8)$$

Variable $x_{n+1} \geq 0$, by means of which the inequality (1.7) is transformed into the equation (1.8), it is called an additional variable. The following theorem shows that inequality (1.7) and equation (1.8) have the same solutions.

Theorem. For each solution $\overline{\mathbf{X}} = (\beta_1, \beta_2, \ldots, \beta_n)$ *inequality (1.7) corresponds to a single solution* $\overline{\mathbf{Y}} = (\beta_1, \beta_2, \ldots, \beta_n, \beta_{n+1})$ *equations (1.8), and, conversely, each solution* $\overline{\mathbf{Y}}$ *equation (1.8) corresponds to a single solution* $\overline{\mathbf{X}}$ *inequalities (1.7).*

Proof. Let be the solution of inequality (1.7). Then there is a relation $a_1\beta_1 + a_2\beta_2 + \ldots + a_n\beta_n \leq b$.

Let's move the left part of this inequality to the right and denote the resulting expression in the right part by β_{n+1}:

$$0 \leq b - (a_1\beta_1 + a_2\beta_2 + \ldots + a_n\beta_n) = \beta_{n+1}.$$

It follows that the solution $\overline{\mathbf{Y}} = (\beta_1, \beta_2, \ldots, \beta_n, \beta_{n+1})$ Indeed, it is a solution to the equation (1.8):

$$a_1\beta_1 + a_2\beta_2 + \ldots + a_n\beta_n + \beta_{n+1} =$$
$$= a_1\beta_1 + a_2\beta_2 + \ldots + a_n\beta_n + b - (a_1\beta_1 + a_2\beta_2 + \ldots + a_n\beta_n) = b.$$

On the contrary, let satisfy equation (1.8), i.e. let the relations take place

$$a_1\beta_1 + a_2\beta_2 + \ldots + a_n\beta_n + \beta_{n+1} = b,\ \beta_{n+1} \geq 0.$$

Then, discarding the number on the left side of this equality $\beta_{n+1} \geq 0$, we get the inequality

$$a_1\beta_1 + a_2\beta_2 + \ldots + a_n\beta_n \leq b.$$

It follows that $\overline{\mathbf{X}} = (\beta_1, \beta_2, \ldots, \beta_n)$ — solving the inequality (2.7).

Now consider a linear inequality with n unknowns

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n \geq b. \qquad (1.9)$$

To convert this inequality into an equation, we subtract an additional variable from its large side $x_{n+1} \geq 0$. As a result, we obtain a linear equation with n + 1 unknowns

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n - x_{n+1} = b. \qquad (1.10)$$

Thus, any inequalities in the constraint system of a linear programming problem can be transformed into equations. That is, if the system of constraints of a linear programming problem contains inequalities, then by introducing into each of them its own non-negative additional variable, it can be transformed into a system of equations. In this case, each additional variable enters the linear function with a coefficient equal to zero.

For example, let the linear programming problem be given as

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n, \qquad (1.11)$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \leq b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_m \end{cases}, \qquad (1.12)$$

$$x_1 \geq 0,\ x_2 \geq 0,\ \ldots,\ x_n \geq 0. \qquad (1.13)$$

In the above–mentioned way, this task can be reduced to a similar form to a task of type (1.4) - (1.6)

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + 0\cdot(x_{n+1} + \ldots + x_{n+m}), \qquad (1.14)$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + x_{n+1} = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + x_{n+2} = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + x_{n+m} = b_m \end{cases}, \qquad (1.15)$$

$$x_1 \geq 0,\ x_2 \geq 0, \ldots,\ x_n \geq 0,\ x_{n+1} \geq 0, \ldots,\ x_{n+m} \geq 0. \qquad (1.16)$$

Now, let's assume that the linear programming problem is given in a different form:

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n, \qquad (1.17)$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b_m \end{cases}, \qquad (1.18)$$

$$x_1 \geq 0,\ x_2 \geq 0, \ldots,\ x_n \geq 0. \qquad (1.19)$$

In this case, this task can also be reduced to a similar type of task (1.4) – (1.6)

$$Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n + 0 \cdot (x_{n+1} + \ldots + x_{n+m}), \qquad (1.20)$$

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n - x_{n+1} = b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n - x_{n+2} = b_2 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n - x_{n+m} = b_m \end{cases}, \qquad (1.21)$$

$$x_1 \geq 0,\ x_2 \geq 0, \ldots,\ x_n \geq 0,\ x_{n+1} \geq 0, \ldots,\ x_{n+m} \geq 0. \qquad (1.22)$$

It should also be noted that the right-hand sides of inequalities and equations of any system of constraints can be considered non-negative, i.e. $b_i \geq 0$ ($i = 1,2,\ldots,m$). If for some k-th inequality or equation $b_k < 0$, then this inequality or equation is multiplied by -1, and the sign of the inequality is reversed.

If for any variable x_j the non-negativity condition is not set ($x_j \geq 0$), that is, there is no restriction on the sign of this variable, then this variable can be represented as the difference between two non-negative variables:

$$x_j = x_k - x_l,\ x_k \geq 0,\ x_l \geq 0.$$

Then you need to substitute this difference instead of a variable x_j in all constraints and in the objective function.

On the other hand, if for any variable x_j instead of a condition $x_j \geq 0$ the condition is set $x_j \geq b_j$, then from this variable to another non-negative variable x_k they switch using formulas

$$x_j = b_j + x_k \text{ и } x_k = x_j - b_j \geq 0.$$

To transform the problem of finding the maximum value of the objective function of a linear programming problem into the problem of finding the minimum value of this function, the objective function should be taken with the opposite sign and vice versa.

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