https://gospodarkainnowacje.pl GOSPODARKA I INNOWACJE

LABORATORIUM WIEDZY Artur Borcuch

Volume: 50 | 2024

Economy and Innovation

ISSN: 2545-0573

For more information contact : editor@gospodarkainnowacje.pl

MATEMATIK INDUKSIYA METODINING NOSTANDART MASALALARDA QO‘LLANILISHI HAQIDA

Sadiqova Sevara

Urganch davlat universiteti 2-bosqich magistranti

A R T I C L E I N F O.

matematik induksiya metodi, matematik induksiya printsipi, tengsizlik, n - tartibli determinant, n - tartibli hosila.

***Abstract*:**

Ushbu maqolada matematik xulosalarni isbotlashning zarur va samarali usuli ko‘rib chiqiladi. Maqolada turli tenglamalar, turli tartibli matritsalarning n - darajasi, funksiyalarning n - tartibli hosilalari, n - tartibli determinantlar, ba’zi I-tur xosmas integrallarni hisoblash matematik induksiya metodidan foydalanish usullari ko‘rsatilgan.

Aytish mumkinki, induksiya metodi bilishning shunday yo‘liki, bunda o‘quvchining fikri birlikdan umumiylikka, xususiy xulosalardan umumiy xulosaga o‘sib boradi. Induktiv xulosa xususiydan umumiyga qarab boradigan xulosadir. Bu metoddan foydalanib biror qonuniyatni ochish yoki qoidani chiqarish uchun o‘qituvchi misollar, masalalar, ko‘rsatmali materiallarni puxtalik bilan tanlaydi. Tushunchalarni matematik induksiya metodi bilan isbotlashda uning asosi hisoblangan matematik induksiya metodini bilish va uni ko‘plab xulosalarini isbotlash metodi hisoblanadi.

“Induksiya” atamasi lotincha ildizlarga ega va so‘zma-so‘z "yo‘l-yo‘riq" deb tarjima qilinadi. Bu atamani matematikaga Djon Vallis 1656-yil "Umumiy arifmetika" asarida kiritgan. “Matematik induksiya” atamasini birinchi marta shotland matematiki Augusta de Morgan 1838-yil Britan ensiklopediyasidagi maqolasida keltirgan.

Matematik induksiya metodining negizida matematik induksiya printspi deb ataladigan arifmetikaning aksiomalari yotadi. Dastlab bu printsp haqida to‘xtalamiz: *agar berilgan natural* n *soniga bog‘liq bo‘lgan* $A(n)$ *tasdiq,*$n = 1$ *da o‘rinli va* $n = k$ *(k - ixtiyoriy natural son) o‘rinli ekanligidan, keyingi qadam* $n = k + 1$ *uchun o‘rinli bo‘lishi isbotlansa, u holda* $A(n)$ *tasdiq ixtyoriy natural* n *uchun o‘rinli deb qaraladi.*

Matematik induksiya metodidan foydalainib isbotlash quyidagicha bo‘ladi:
1) $A(n)$ tasdiq $n = 1$ da to‘g‘ri bo‘lsa, ya’ni $A(1)$ tasdiqning to‘g‘riligi tekshirib ko‘riladi. Bu matematik iduksiyaning bazisi deyiladi
2) $A(n)$ tasdiq da to‘g‘ri deb qaralib, $A(k + 1)$ tasdiqning o‘rinli bo‘lishi isbotlanadi, ya’ni $A(k)$ Ю $A(k + 1)$ bo‘lishi isbotlanadi. Bu induksion qadam deyiladi.

Induksiya bosqichidagi $A(n)$ tasdiq esa induksiya gipotezasi deyiladi. Demak, xulosani matematik induksiya metodida isbotlash uchun ikkita mulohazani isbotlash kerak.

Matematik induksiya usulidan foydalanib, n - tartibli detirminantni hisoblash va turli tartibli matritsaning darajasini toppish mumkin ([2] ga qarang).

1-misol. Quyidagi n - tartibli detirminantni hisoblang, "n О Г :

$$D_n = \begin{vmatrix} 2 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 2 \end{vmatrix}$$

Yechish: Dastlab $D_1 = |2| = 2,\ D_2 = \begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = 3,\ D_3 = \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 4, \dots$ larni hisoblaymiz. Ushbu qonuniyat orqali biz quyidagi xulosaga kelamiz:

$$D_n = \begin{vmatrix} 2 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \dots & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 2 \end{vmatrix} = n + 1. \qquad (1)$$

Endi (1) formulani matematik induksiya metodi bilan isbotlaymiz.

Induksiya bazisi: $A(1)$ tasdiq to‘g‘ri, ya’ni $D_1 = |2| = 2$.

Induksiya qadami: Faraz qilamizki, $A(k)$ tasdiqni har qanday natural k son uchun to‘g‘ri deb qaraymiz, ya’ni $D_k = k + 1$. Endi $A(k+1)$ ning to‘g‘riligini isbotlashimiz kerak. Agar bi$n = k$ rinchi satr bo‘yicha D_{k+1} detirminantni hisoblasak, ya’ni

$$D_{k+1} = 2D_k - D_{k-1} = 2(k+1) - k = k + 2$$

Ekani kelib chiqadi. Bu esa $A(n)$ tasdiqni to‘g‘rigini bildiradi, ya’ni (1) formulani to‘g‘riligi isbotlandi, Demak, $D_n = n + 1$.

2-misol. $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ matritsaning n - darajasini toping.

Yechish:

$$A^1 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = A$$

$$A^2 = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} = \begin{pmatrix} 2^2 & 2^2 - 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^2 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} 4 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 8 & 7 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{pmatrix} = \begin{pmatrix} 2^3 & 2^3 - 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^3 \end{pmatrix}$$

Oxirgi tenglikdan quyidagi xulosaga kelamiz

$$A^n = \begin{pmatrix} 2^n & 2^n - 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix}. \qquad (2)$$

Endi bu tasdiqning to‘g‘riligini matematik induksiya metodi yordamida isbotlaymiz.

Induksiya bazisi: $A(1)$ tasdiq to‘g‘ri.

Induksiya qadami: Faraz qilamizki, $A(k)$ tenglik har qanday natural k son uchun to‘g‘ri, ya’ni

$$A^k = \begin{pmatrix} 2^k & 2^k - 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^k \end{pmatrix}.$$

Endi $A(k+1)$ ning to‘g‘riligini ko‘rsatamiz.

$$A^{k+1} = A^k \cdot A = \begin{pmatrix} 2^k & 2^k - 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^k \end{pmatrix}\begin{pmatrix} 2 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 2^{k+1} & 2^{k+1} - 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^{k+1} \end{pmatrix}.$$

Shunday qilib, har qanday n natural son uchun (2) tasdiqni to‘g‘rigi isbotlandi.

3-misol. $A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$ matritsaning $n-$ darajasini toping.

Yechish:

$$A^1 = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = A$$

$$A^2 = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^2 & 2a \\ 0 & a^2 \end{pmatrix}$$

$$A^3 = A^2 \cdot A = \begin{pmatrix} a^2 & 2a \\ 0 & a^2 \end{pmatrix}\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^3 & 3a^2 \\ 0 & a^3 \end{pmatrix}$$

$$A^4 = A^3 \cdot A = \begin{pmatrix} a^3 & 3a^2 \\ 0 & a^3 \end{pmatrix}\begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^4 & 4a^3 \\ 0 & a^4 \end{pmatrix}$$

Oxirgi tenglikdan quyidagi xulosaga kelamiz

$$A^n = \begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}. \qquad (3)$$

Endi bu tasdiqning to‘g‘riligini matematik induksiya metodi yordamida isbotlaymiz.

Induksiya bazisi: $A(1)$ tasdiq to‘g‘ri.

Induksiya qadami: Faraz qilamizki, $A(k)$ tenglik har qanday natural k son uchun to‘g‘ri bo‘lsin, ya’ni

$$A^k = \begin{pmatrix} a^k & ka^{k-1} \\ 0 & a^k \end{pmatrix}.$$

Endi $A(k+1)$ ning to‘g‘riligini ko‘rsatamiz.

$$A^{k+1} = A^k \cdot A = \begin{pmatrix} a^k & ka^{k-1} \\ 0 & a^k \end{pmatrix} \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} = \begin{pmatrix} a^{k+1} & (k+1)a^k \\ 0 & a^{k+1} \end{pmatrix}.$$

Bu esa har qanday n natural son uchun (3) tasdiqni to‘g‘rigi isbotlandi.

Bundan tashqari xosmas integralni hisoblashda matematik induksiya metodidan ham foydalanish mumkin ([3] ga qarang).

4-misol. $\int_0^\infty x^n e^{-x} dx$ xosmas integralni hisoblang.

Yechish: Agar $n = 1$ bo‘lsa, u holda $I_1 = \int_0^\infty x e^{-x} dx$ bo‘ladi. Bu integralni bo‘laklab integrallash usuli bilan integralni hisoblaymiz

$$\int_0^\infty x e^{-x} dx = \left(-xe^{-x} - e^{-x}\right)\Big|_0^\infty = 1 = 1!$$

$n = 2$ bo‘lsa, $I_2 = \int_0^\infty x^2 e^{-x} dx = \left(-x^2 e^{-x}\right)\Big|_0^\infty + 2\int_0^\infty x e^{-x} dx = 2 = 2!.$

$n = 3$ bo‘lsa, $I_3 = \int_0^\infty x^3 e^{-x} dx = 6 = 3!.$

Oxirgi tenglamalardan biz quyidagi xulosaga kelamiz.

$$I_n = \int_0^\infty x^n e^{-x} dx = n!. \tag{4}$$

Bu (4) tasdiqning to‘g‘riligini matematik induksiya metodi yordamida isbotlaymiz.

Induksiya bazisi: $I(1)$ tasdiq to‘g‘ri. $\int_0^\infty x e^{-x} dx = \left(-xe^{-x} - e^{-x}\right)\Big|_0^\infty = 1$

Induksiya qadami: Faraz qilamizki, $I(k)$ tenglik har qanday k natural son uchun to‘g‘ri bo‘lsin, ya’ni

$$\int_0^\infty x^k e^{-x} dx = k!$$

Endi $I(k+1)$ ning to‘g‘riligini ko‘rsatamiz.

$$\int_0^{\infty} x^{k+1}e^{-x}dx = (k+1)!.$$

Haqiqattan ham,

$$\int_0^{\infty} x^{k+1}e^{-x}dx = \left(-x^{k+1}e^{-x}\right)\Big|_0^{\infty} + (k+1)\int_0^{\infty} xe^{-x}dx = (k+1)k! = (k+1)!$$

Demak, (4) tasdiq har qanday n natural son uchun to‘g‘rigi isbotlandi.

5-misol. $\int_0^{\infty} x^n 2^{-x}dx$ xosmas integralni hisoblang.

Yechish: Agar $n = 1$ bo‘lsa, u holda $I_1 = \int_0^{\infty} x\cdot 2^{-x}dx$ bo‘ladi. Bu integralni bo‘laklab integrallash usuli bilan integralni hisoblaymiz

$$\int_0^{\infty} x\,2^{-x}dx = \frac{x\cdot 2^{-x}}{\ln 2}\Bigg|_0^{\infty} - \frac{1}{\ln 2}\int_0^{\infty} 2^{-x}dx = \frac{1!}{(\ln 2)^2}$$

$n = 2$ bo‘lsa, $I_2 = \int_0^{\infty} x^2 2^{-x}dx = \dfrac{\left(x^2 2^{-x}\right)}{\ln 2}\Bigg|_0^{\infty} + \dfrac{2}{\ln 2}\int_0^{\infty} x\,2^{-x}dx = \dfrac{2!}{(\ln 2)^3}.$

$n = 3$ bo‘lsa, $I_3 = \int_0^{\infty} x^3 2^{-x}dx = \dfrac{3!}{(\ln 2)^4}.$

Oxirgi tenglamalardan biz quyidagi xulosaga kelamiz.

$$I_n = \int_0^{\infty} x^n 2^{-x}dx = \frac{n!}{(\ln 2)^{n+1}}. \qquad (4)$$

Bu (4) tasdiqning to‘g‘riligini matematik induksiya metodi yordamida isbotlaymiz.

Induksiya bazisi: $I(1)$ tasdiq to‘g‘ri.

$$\int_0^{\infty} x\,2^{-x}dx = \frac{x\cdot 2^{-x}}{\ln 2}\Bigg|_0^{\infty} - \frac{1}{\ln 2}\int_0^{\infty} 2^{-x}dx = \frac{1!}{(\ln 2)^2}$$

Induksiya qadami: Faraz qilamizki, $I(k)$ tenglik har qanday k natural son uchun to‘g‘ri bo‘lsin, ya’ni

$$\int_0^{\infty} x^k 2^{-x} dx = \frac{k!}{(\ln 2)^{k+1}}$$

Endi $I(k+1)$ ning to‘g‘riligini ko‘rsatamiz.

$$\int_0^{\infty} x^{k+1} 2^{-x} dx = \frac{(k+1)!}{(\ln 2)^{k+2}}.$$

Haqiqattan ham,

$$\int_0^{\infty} x^{k+1} 2^{-x} dx = \left.\frac{x^{k+1} 2^{-x}}{\ln 2}\right|_0^{\infty} + \frac{(k+1)}{\ln 2}\int_0^{\infty} x^k 2^{-x} dx = \frac{(k+1)k!}{(\ln 2)^{k+2}} = \frac{(k+1)!}{(\ln 2)^{k+2}}$$

Demak, (4) tasdiq har qanday n natural son uchun to‘g‘rigi isbotlandi.

Ayrim funksiyalarning n - tartibli hosilasini hisoblash uchun matematik induksiya metodidan foydalanish mumkin.

6-misol. $f(x) = \frac{1}{ax+b}$ funksiyaning n - tartibli hosilasini hisoblang.

Yechish: 1-tartibli hosilasi $f'(x) = -\frac{a}{(ax+b)^2}$, 2-tartibli hosila $f''(x) = \frac{2a^2}{(ax+b)^3}$, shu kabi $f'''(x) = -\frac{6a^3}{(ax+b)^4}, \ldots$.

Ushbu qonuniyat orqali biz quyidagi formulaga kelamiz:

$$f^{(n)} = \frac{(-1)^n a^n n!}{(ax+b)^{n+1}}, \qquad (5)$$

Bu formulani to‘g‘riligini matematik induksiya usuli bilan isbotlaylik.

Induksiya bazisi: $n = 1$ da tasdiq to‘g‘ri, ya’ni

$$f^1 = \frac{(-1)^1 a^1 1!}{(ax+b)^{1+1}} = -\frac{a}{(ax+b)^2}$$

Induksiya qadami: Faraz qilamizki, (5) tenglik har qanday k natural son uchun to‘g‘ri bo‘lsin, ya’ni

$$y^{(k)} = \frac{(-1)^k a^k k!}{(ax+b)^{k+1}}$$

Endi $n = k+1$ uchun (5) ning to‘g‘riligini ko‘rsatamiz.

$$y^{(k+1)} = \frac{(-1)^{k+1} a^{k+1} (k+1)!}{(ax+b)^{k+2}}.$$

Haqiqatan ham,

$$y^{(k+1)} = \left(y^{(k)}\right)' = \left(\frac{(-1)^k a^k k!}{(ax+b)^{k+1}}\right)' = \frac{(-1)^{k+1} a^{k+1} (k+1)!}{(ax+b)^{k+2}}.$$

Demak, (5) formula har qanday n natural son uchun to‘g‘rigi isbotlandi.

7-misol. $y = \ln x$ funksiyaning n - tartibli hosilasini toping.

Yechish: 1-tartibli hosilasi $f'(x) = \frac{1}{x}$, 2-tartibli hosila $f''(x) = -\frac{1}{x^2}$, shu kabi $f'''(x) = \frac{2}{x^3} = \frac{2!}{x^3}, f''''(x) = -\frac{6}{x^4} = -\frac{3!}{x^4} \ldots$.

Ushbu qonuniyat orqali biz quyidagi formulaga kelamiz:

$$f^{(n)} = \frac{(-1)^{n+1} (n-1)!}{x^n}, \tag{5}$$

Bu formulani to‘g‘riligini matematik induksiya usuli bilan isbotlaylik.

Induksiya bazisi: $n = 1$ da tasdiq to‘g‘ri, ya’ni

$$f^1 = \frac{(-1)^{1+1} (1-1)!}{x^1} = \frac{1}{x}$$

Induksiya qadami: Faraz qilamizki, (5) tenglik har qanday k natural son uchun to‘g‘ri bo‘lsin, ya’ni

$$f^{(k)} = \frac{(-1)^{k+1} (k-1)!}{x^k},$$

Endi $n = k + 1$ uchun (5) ning to‘g‘riligini ko‘rsatamiz.

$$f^{(k+1)} = \frac{(-1)^{k+2} (k)!}{x^{k+1}}.$$

Haqiqatan ham,

$$y^{(k+1)} = \left(y^{(k)}\right)' = \left(\frac{(-1)^{k+1} (k-1)!}{x^k}\right)' = \frac{(-1)^{k+1} (k-1)! k}{x^{k+1}} = \frac{(-1)^{k+1} k!}{x^{k+1}}.$$

Demak, (5) formula har qanday n natural son uchun to‘g‘rigi isbotlandi.

Literature

1. *Соминский И.С., Головина Л.И, Яглом И.М., О математической индукции. - М:*

Наука. -144 с.
2. *Винберг Э.Б. Курс алгебры. - 2-е изд.,стер. - М :МЦНМО, 2013. - 590 с.*
3. *Ильин В.А., Садовничий В.А., Сендов Б.Х Математический анализ. - М: МГУ, 1985. - 662 с.*
4. *Abdurahmonov B. Matematik induksiya metodi. Toshkent, 2008 y.*