

TANLANMANING IKKINCHI DARAJALI REGRESIYA TENGLAMASINI TENLAMASINI KICHIK KVADRATLAR USULIDA ANIQLASH

Mirzakarimov Ergashboy Mirzaboyevich

Dotsent, Farg'ona politexnika instituti, Farg'ona, Uzbekiston

A R T I C L E I N F O.

Kalit so'zlar: regressiya, kriteriya, model, adekvat, Maple, seq, add.

Annotatsiya

Hozirgi vaqtida har qanday jiddiy statistik hisob-kitoblar, qoida tariqasida, kompyuterlarda va bиринчи navbatda, shaxsiy kompyuterlarda amalga oshiriladi. Ushbu maqolada Maple dasturidan foydalanib muxandslik va iqtisodiyot masalalarining tajriba natijalari bo'yicha tuzilgan matematik modellarning sifat va samatadorligi hamda raqamli usullardan foydalanib tahlil va qaror qabul qilishda axamiyatli ekanligi ko'rsatilgan.

<http://www.gospodarkainnowacje.pl> © 2024 LWAB.

KIRISH. Statistik ma'lumotlarni qayta ishlashda regression taxlil bo'yicha tajriba natijalarining chiziqsiz-ikkinci darajali regressiya tenglamasini matematik modeli eng kichik kvadratlar usulidan foydalanib tuzish va bu modelni sifatini ko'rsatamiz[5,8,9].

ASOSIY QISM. 1. Tanlanmada Y ning X ga bog'lanishining ikkinchi darajali regressiya tenglamasini aniqlash.

Ikkinch darajali regressiya tenglamasini topishni quyidagi misol orqali izohlaymiz. Soddarooq bo'lishi uchun kichikroq jadval, hamda chiziqli bo'limgan eng ommalashgan holi kvadrat uchhad ko'rinishi bilan chegaralanamiz.

Quyidagi korrelyasion jadvalda keltirilgan ma'lumotlar bo'yicha $y = ax^2 + bx + c$ regressiya tenglamasini eng kichik kvadratlar usuli yordamida topamiz.

1-jadval

$y \setminus x$	2	3	5	n_y
25	20			20
45		30	1	31
110		1	48	49
n_x	20	31	49	$N=100$

Buning uchun a, b, c parametrlarni

$$F(a, b, c) = \sum \left(y_{x_i} - \bar{y}_{x_i} \right)^2 n_{x_i} = \sum \left(ax_i^2 + bx_i + c - \bar{y}_{x_i} \right)^2 n_{x_i}$$

Kielce: Laboratorium Wiedzy Artur Borcuch

farqlarning kvadratlari minimal bo‘ladigan qilib tanlab olish imkonini beruvchi quyidagi tenglamalar sistemasini hosil qilamiz:

$$\frac{\partial F(a,b,c)}{\partial a} = 2 \sum \left(ax_i^2 + bx_i + c - \bar{y}_{x_i} \right) x_i^2 n_{x_i} = 0$$

$$\frac{\partial F(a,b,c)}{\partial b} = 2 \sum \left(ax_i^2 + bx_i + c - \bar{y}_{x_i} \right) x_i n_{x_i} = 0$$

$$\frac{\partial F(a,b,c)}{\partial c} = 2 \sum \left(ax_i^2 + bx_i + c - \bar{y}_{x_i} \right) n_{x_i} = 0$$

bu sistemadan:

$$\begin{cases} \left(\sum n_x x^4 \right) a + \left(\sum n_x x^3 \right) b + \left(\sum n_x x^2 \right) c = \sum n_x \bar{y}_x x^2 \\ \left(\sum n_x x^3 \right) a + \left(\sum n_x x^2 \right) b + \left(\sum n_x x \right) c = \sum n_x \bar{y}_x x \\ \left(\sum n_x x^2 \right) a + \left(\sum n_x x \right) b + nc = \sum n_x \bar{y}_x \end{cases} \quad (*)$$

Bu sistemadagi yig‘indilarni quyidagicha topamiz:

1-jadval asosida shartli o‘rta qiymatlarni topamiz.

$$\bar{y}_2 = \frac{25 \cdot 20}{20} = 25$$

$$\bar{y}_3 = \frac{45 \cdot 30 + 110 \cdot 1}{31} = 47,1$$

$$\bar{y}_5 = \frac{45 \cdot 1 + 110 \cdot 48}{49} = 108,67$$

2-jadval

x	n_x	\bar{y}_x	$n_x x$	$n_x x^2$	$n_x x^3$	$n_x x^4$	$n_x \bar{y}_x$	$n_x \bar{y}_x x$	$n_x \bar{y}_x x^2$
2	20	25	40	80	160	320	500	1000	2000
3	31	47,1	93	279	837	2511	4380	13140	13141
5	49	108,67	245	12285	6125	30625	5325	26625	133121
Σ	100		378	1584	7122	33456	7285	32004	148262

2-jadval oxirida turgan yig‘indilarni (*) sistemaga qo‘yib, quyidagi sistemanı hosil qilamiz:

$$\begin{cases} 33456a + 7122b + 1584c = 148262 \\ 7122a + 1584b + 378c = 32004 \\ 1584a + 378b + 100c = 7285 \end{cases}$$

Sistemanı echib, $a=2.94$, $b=7.27$, $c=-1.25$ qiymatlarni topamiz va bu qiymatlarni regressiya tenglamasi:

$$\bar{y}_x = ax^2 + bx + c$$

ga qo‘yib,

$$\bar{y}_x = 2.94x^2 + 7.27x - 1.25$$

regressiya tenglamasiga ega bo‘lamiz.

1. Berilgan korrelasion jadval asosida Y ning X ga regressiya chizig‘i $\bar{y}_x = ax^2 + bx + c$ ning tenglamasini topishda kichik kvadratlar usulida tuzilgan sistema koeffisientlarini ko‘paytmalar usulida topishning Maple dasturini tuzamiz.

Maple dasturi:

> restart;with(stats):

1) 4-korrelasion jadval asosida X va Y larini kiritish:

> X:=Vector([2,3,5]);

$$X := \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

$$Y := \begin{bmatrix} 158 \\ 164 \\ 170 \\ 176 \\ 182 \end{bmatrix}$$

> Y:=Vector([158,164,170,176,182]);

2) korrelasion jadval asosida n_x va n_{xy} chastotalarni kiritish:

$$nx := \begin{bmatrix} 20 \\ 31 \\ 49 \end{bmatrix}$$

> nx:=Vector([20,31,49]);

> nxy:=matrix([[20,0,0],[0,30,1],[0,1,48]]);

$$nxy := \begin{bmatrix} 20 & 0 & 0 \\ 0 & 30 & 1 \\ 0 & 1 & 48 \end{bmatrix}$$

3) korrelasiyon jadval asosida shartli o'rta qiymatlarni hisoblash:

> $Yx[1]:=(Y[1]*nxy[1,1]+Y[2]*nxy[2,1]+Y[3]*nxy[3,1])/nx[1];$

$Yx_1 := 25$

> $Yx[2]:=(Y[1]*nxy[1,2]+Y[2]*nxy[2,2]+Y[3]*nxy[3,2])/nx[2];$

$Yx_2 := \frac{1460}{31}$

> evalf(%,.4); 47.10

> $Yx[3]:=(Y[1]*nxy[1,3]+Y[2]*nxy[2,3]+Y[3]*nxy[3,3])/nx[3];$

$Yx_3 := \frac{5325}{49}$

> evalf(%,.4); 108.7

4) korrelasiyon jadval asosida X ning qiymatlar soni n va tanlanma xajmi N qiymatlarni kiritish:

> **n:=3:N:=100:**

5) 2-jadvalning qiymatlarni ko'paytmalar usulidagi hisoblash:

> $Sx:=add(X[k]*nx[k],k=1..n); Sx := 378$

> $Sxx:=add(nx[k]*X[k]^2,k=1..n); Sxx := 1584$

> $Sxxx:=add(nx[k]*X[k]^3,k=1..n); Sxxx := 7122$

> $Sxxxx:=add(nx[k]*X[k]^4,k=1..n); Sxxxx := 33456$

> $SYx:=add(nx[k]*Yx[k],k=1..n); SYx := 7285$

> $SxYx:=add(nx[k]*X[k]*Yx[k],k=1..n); SxYx := 32005$

> $SxxYx:=add(nx[k]*X[k]^2*Yx[k],k=1..n); SxxYx := 148265$

6) kichik kvadratlar usulida tuzilgan sistemani yechish:

> abc:=solve([a*Sxxxx+b*Sxxx+c*Sxx=SxxYx,

a*Sxxx+b*Sxx+c*Sx=SxYx,

a*Sxx+b*Sx+c*N=SYx],{a,b,c});

$abc := \left\{ a = \frac{26405}{9114}, b = \frac{69365}{9114}, c = -\frac{2750}{1519} \right\}$

> evalf(%,.4); {b = 7.611, c = -1.810, a = 2.897}

7) regressiya egri chizig‘ining tenglamasini yozish:

> **y:=abc[1]*x^2+abc[2]*x+abc[3];**

$$y := x^2 a + x b + c = \frac{26405}{9114} x^2 + \frac{69365}{9114} x - \frac{2750}{1519}$$

> **y:=evalf(%,.4);**

$$y := x^2 a + x b + c = 2.897 x^2 + 7.611 x - 1.810$$

2. Berilgan korrelasion jadval asosida Y ning X ga regressiya chizig‘i $\bar{y}_x = ax^2 + bx + c$ ning tenglamasini topishda **fit** asfunksiyasidan foydalanib Maple dasturini tuzamiz.

Maple dasturi:

> **restart; with(stats);**

1) korrelasion jadval asosida X va Y larining qiymatlarini chastotalari bilan satr bo‘yicha kiritish:

> **W:=[[Weight(2,20),Weight(3,30),Weight(5,1), Weight(3,1), Weight(5,48)], [Weight(25,20),Weight(45,30),Weight(45,1),Weight(110,1),Weight(110,48)]];**

**W:=[[Weight(2, 20), Weight(3, 30), Weight(5, 1), Weight(3, 1),
Weight(5, 48)], [Weight(25, 20), Weight(45, 30), Weight(45, 1),
Weight(110, 1), Weight(110, 48)]]**

2) X va Y larining qiymatlari bo‘yicha (x,y) larni koordinatalar sistemasida aniqlash:

> **statplots[scatterplot](W[1],W[2],color=blue, symbol=BOX, symbolsize=20);** (1-rasm)

3) regressiya eg‘ri chizig‘ining tenglamasini aniqlash:

> **x:=vector(transform[statvalue](W[1]));**

$$x := \begin{bmatrix} 2 & 3 & 5 & 3 & 5 \end{bmatrix}$$

> **y:=vector(transform[statvalue](W[2]));**

$$y := \begin{bmatrix} 25 & 45 & 45 & 110 & 110 \end{bmatrix}$$

> **fit[leastsquare][[x,y],y=a*x^2+b*x+c]](W);**

$$y = \frac{26405}{9114} x^2 + \frac{69365}{9114} x - \frac{2750}{1519}$$

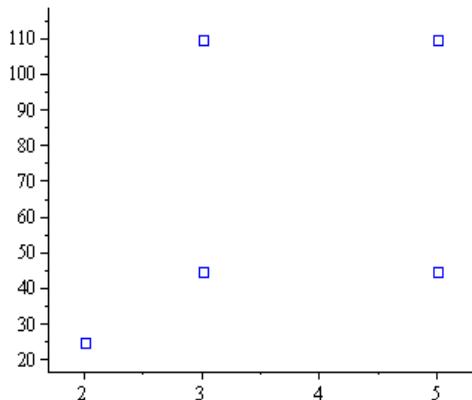
> **evalf(%,.5);**

$$y = 2.8972 x^2 + 7.6108 x - 1.8104$$

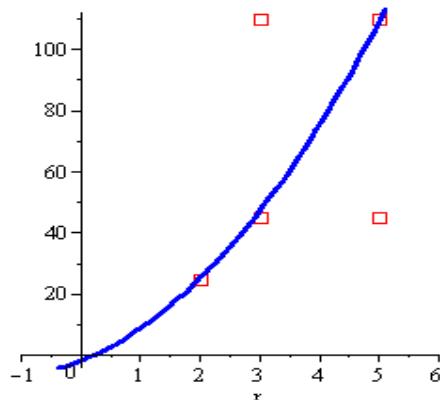
- 4) regressiya eg‘ri chizig‘ini qurish:

> with(plots):

```
> plot([[x[i],y[i],i=1..5],2.8972*x^2+7.6108*x-1.8104],      x=-1..6,-4..112,style=[point,line],
color=[red,blue],symbol=BOX,symbolsize=25,
view=[-1..6,-4..112],thickness=3); (2-rasm)
```



1-rasm.



2-rasm.

XULOSA. Demak, kichik kvadratlar usuli asosida topilgan ikkinchi darajali bog'lanish-modeli adikvat bo'lib, uning barcha koeffitsentlari qiymatdor ekanligini topdik. Berilgan tajriba natijalari bo'yicha xulosa va qaror qabul qilish uchun topilgan tajriba natijalari bo'yicha bog'lanish modelni tuzish va samaradorligini aniqlashda ushbu Maple dasturidan foydalanib aniq, tez va sifatli natijalarni olish mumkinligini ko'rdik.

Adabiyotlar

1. Jamshid Faizullayev, Ergashboy Mirzakarimov, Jamshid Mamayusupov, Bobur Tillaboyev and Guljahan Tillaboyeva. (2024). Methods of teaching fundamental sciences based on the integration of information and pedagogical technologies. E3S Web of Conferences (Vol.538, p.05012). 14 June 2024. <https://www.e3s-conferences.org/articles/e3sconf/abs/2024/68/contents/contents>. DOI: <https://doi.org/10.1051/e3sconf/202453805012>.
2. Кремер Н. И., (2001), «Теория вероятностей и математическая статистика». Учебное пособие. Москва.
3. Матросов А.В. (2001). Maple 6. Решение задач высшей математики и механики. СПб.: БХВ-Петербург.
4. Mirzakarimov, E.M., (2021). Maple tizimida to'g'ri to'rtburchakli membrananing erkin tebranishini aniqlash. *Scientific Bulletin of Namangan State University* 1(8), 3-9.
5. Mirzakarimov, E.M., (2022). To'g'riburchakli membrananing erkin tebranishini boshlang'ich shartlar bo'yicha aniqlashda maple tizimidan foydalanish. *Scientific Bulletin of Namangan State University* 1(1), 53-61.
6. Mirzakarimov, E.M., & Fayzullaev, J.S. (2020). Improving the quality and efficiency of teaching by developing students' mathematical competence using the animation method of adding vectors to the plane using the maple system. *Scientific Bulletin of Namangan State University*, 2(9), 336-342.
7. Mirzakarimov, E.M., & Faizullaev, J.I. (2019). Method of teaching the integration of information and

- educational technologies in a heterogeneous parabolic equation. *Scientific Bulletin of Namangan State University*, 1(5), 13-17.
8. Mirzaboevich, M.E., & ugli, E. M.I. (2021). Using Maple Programs in Higher Mathematics Triangle Problem Constructed on Vectors in Space. *Central asian journal of mathematical theory and computer sciences*, 2(11), 44-50. Retrieved from <http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/123>
 9. Мирзабоевич, М.Э., & Исломиджонович, Ф.Д. (2021). Выполнять Линейные Операции Над Векторами В Пространстве В Системе Maple. *Central asian journal of mathematical theory and computer sciences*, 2(12), 10-16.
<http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/137>
 10. Мирзабоевич, М.Э., (2022). Использовать Систему Maple Для Определения Свободных Колебаний Прямоугольной Мембранны При Начальных Условиях. *Central Asian Journal Of Mathematical Theory And Computer Sciences*, 3(1), 9-18. <https://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/148>
 11. Mirzakarimov E.M. (2018). Oliy matematika fanidan laboratoriya ishlarini Maple dasturida bajarish. T.: Tafakkur bo'stoni.
 12. Mirzakarimov , E. ., & Fayzullayev, D. . (2023). Maple tizimida kompleks sonlarlarning geometrik tasviri. *Евразийский журнал математической теории и компьютерных наук*, 3(5), 38–45. <https://www.in-academy.uz /index.php/EJMTCS/article/view/14347>
 13. Mirzakarimov , E. ., & Fayzullayev, D. . (2023). Algebraik shakildagi kompleks sonlar ustida amallarni maple tizimida bajarish. *Евразийский журнал математической теории и компьютерных наук*, 3(5), 30–37.
<https://in-academy.uz /index.php/EJMTCS/article/view/14346>
 14. E.M.Mirzakarimov, & J.I.Fayzullayev.(2024). “Geometric representation of complex numbers in the maple system” *Gospodarka i Innowacje*, (45), 70-78,
https://gospodarkainnowacje-pl.openconference.us/index.php/issue_view_32/article/view/2373/2195,
 15. E.M.Mirzakarimov, & J.I.Fayzullayev.(2024), “Linear operations on complex numbers in the maple system” *Gospodarka i Innowacje*, (45), 79-86
https://www.gospodarkainnowacje.pl/index.php/issue_view_32/article/view/2374/2196
 16. Mirzakarimov Ergashboy Mirzaboyevich, Fayzullayev Djamshid Ismoiljonovich “Solving Differential Equations Using the Operational Method In the Maple System”, Excellencia: International Multi-Disciplinary Journal Of Education, 2024 Volume 02, 212-216, <https://multijournals.org/index.php/excel lencia-imje/article/view/514/590>.