TO'G'RI CHIZIQLI REGRESSIYA TENLAMASINI KICHIK KVADRATLAR USULIDA ANIQLASH

Mirzakarimov Ergashboy Mirzaboyevich

Dotsent, Farg'ona politexnika instituti, Farg'ona, Uzbekiston

A R T I C L E I N F O.	**Annotatsiya**
Kalit so'zlar: regressiya, kriteriya, model, adekvat, Maple, seq, add.	Hozirgi vaqtda har qanday jiddiy statistik hisob-kitoblar, qoida tariqasida, kompyuterlarda va birinchi navbatda, shaxsiy kompyuterlarda amalga oshiriladi. Ushbu maqolada Maple dasturidan foydalanib muxandislik va iqtisodiyot masalalarning tajriba natijalari bo'yicha tuzilgan matematik modellarning sifat va samatadorligi hamda raqamli usullardan foydalanib tahlil va qaror qabul qilishda axamiyatli ekanligi ko'rsatilgan.

KIRISH. Statistik ma'lumotlarni qayta ishlashda regression taxlil bo'yicha tajriba natijalarining matematik modeli-chiziqli regressiya tenglamasini eng kichik kvadratlar usulidan foydalanib tuzish va bu modellarni sifatini hamda korrelyasion nisbatini hisoblash masalalari ko'rilgan[5,8,9].

ASOSIY QISM. 1. Y ning X ga regressiya to'g'ri chizig'ining tanlanma tenglamasini aniqlash.

X ning qismiy intervallari va Y ning qismiy intervallari kesishgan qismga tushuvchi $(x_i; y_j)$ qiymatlarni sanab, (Bunda intervallarning chegaralariga to'g'ri kelgan qiymatlarni faqat oldingi intervallarga tushadi deb sanaymiz).

Qatorlar bo'yicha chastotalarni jamlab, n_y larni topamiz va oxirgi ustunga yozamiz.

Ustunlar bo'yicha chastotalarni jamlab, n_x larni topamiz va oxirgi qatorga yozamiz.

n_x larning yig'indisi ham , n_y larning yig'indisi ham tanlanma hajmi $n=20$ ga teng bo'ladi.

1 - jadval

Y \ X	$h_1 = 4$	90 – 94	94 - 98	98 - 102	102 - 106	
$h_2 = 6$	Y \ X	$X_1 = 92$	$X_2 = 96$	$X_3 = 100$	$X_4 = 104$	n_y
155 – 161	$Y_1 = 158$	1	1			2
161 – 167	$Y_2 = 164$	1	4	1		6
167 – 173	$Y_3 = 170$		2	5	1	8
173 – 179	$Y_4 = 176$			1	2	3
179 – 185	$Y_5 = 182$				1	1
	n_x	2	7	7	4	$n = 20$

Bu jadvaldan quyidagi 2- jadvalni ajratamiz:

2-jadval

Y/X	92	96	100	104	n_y
158	1	1			2
164	1	4	1		6
170		2	5	1	8
176			1	2	3
182				1	1
n_x	2	7	7	4	$N=20$
$\overline{y}_x$	161	164.8	170	176	

To'g'ri chiziqli bog'laninsh regressiya tenglamasini topish uchun berilgan jadvaldagi ma'lumotlar bo'yicha y ning x ga regressiya to'g'ri chizig'ining tanlanma tenglamasini

$$y_x = ax + b \qquad (1)$$

ko'rinishda izlaylik.

Buning uchun a, b parametrlarni topish uchun, quyidagi

$$F(a,b) = \sum \left(y_{xi} - \overline{y}_{xi} \right)^2 n_{xi} = \sum \left(ax_i + b - \overline{y}_{xi} \right)^2 n_{xi}$$

farqlarning kvadratlari minimal bo'ladigan qilib tanlab olish imkonini beruvchi quyidagi tenglamalar sistemasini hosil qilamiz:

$$\frac{\partial F(a,b)}{\partial a} = 2\sum \left(ax_i + b - \overline{y}_{x_i} \right) x_i n_{x_i} = 0$$

$$\frac{\partial F(a,b)}{\partial b} = 2\sum \left(ax_i + b - \overline{y}_{x_i} \right) n_{x_i} = 0$$

bu sistemadan:

$$\left. \begin{array}{l} \left(\sum n_x x^2 \right) a + \left(\sum n_x x \right) b = \sum n_x x \cdot \overline{y}_x \\ \left(\sum n_x x \right) a + nb = \sum n_x \overline{y}_x \end{array} \right\} \qquad (2)$$

Bu sistemani yechib, a, b - parametrlarni aniqlovchi munosabatlarga ega bo'lamiz.

$$a = \frac{n\sum n_x x \cdot \overline{y}_x - \sum n_x x \cdot \sum n_x \overline{y}_x}{n\sum n_x x^2 - \left(\sum n_x x \right)^2} \qquad (3)$$

$$b = \frac{\sum n_x \overline{y}_x \cdot \sum n_x x^2 - \sum n_x x \cdot \sum n_x x \overline{y}_x}{n \sum n_x x^2 - \left(\sum n_x x\right)^2} \qquad (4)$$

Topilgan 2-korrelasion jadvaldagi ma'lumotlar asosida quyidagi 3-jadvalni ko'paytmalar usulida tuzamiz:

3-jadval

n_x	x	$\overline{y}_x$	$n_x x$	$n_x x^2$	$n_x \overline{y}_x$	$n_x x \overline{y}_x$
2	92	161	164	16928	316	29624
7	96	164,8	672	64512	1154	40746
7	100	170	700	70000	1190	119000
4	104	176	416	43264	704	73216
20			1972	1947004	3370	332586

3-jadvaldagi oxirgi qatorga yozilgan yig'indilarni (3) va (4) ga qo'yib,

$$a = \frac{20 \cdot 332586 - 1972 \cdot 3370}{20 \cdot 19477004 - 1972^2} = 1{,}3 \, ,$$

$$b = \frac{3370 \cdot 194704 - 1972 \cdot 332586}{20 \cdot 194704 - 1972^2} = 40{,}8$$

topilgan a va b larning qiymatlari asosida izlanayotgan regressiya tenglamasi:

$y_x = ax + b = 1.3x + 40.8$

bu tenglama bo'yicha hisoblanadigan y_{xi} qiymatlar kuzatilgan $\overline{y}_{xi}$ qiymatalarga

qanchalik mos kelishini topish uchun, y_{xi} va $\overline{y}_{xi}$ qiymatlari orasidagi farqlarni aniqlash maqsadida quyidagi jadvalni tuzamiz:

4-jadval

x_i	y_{xi}	$\overline{y}_{xi}$	$y_{xi} - \overline{y}_{xi}$
92	160.4	161	-0.6
96	165.4	164.8	0.8
100	170.8	170	0.8
104	176	176	0

Jadvaldagi farqlar bog'lanishining aniqligini ifodalab beradi. Bu jadvaldan ko'rinadiki chetlanishlarning hammasi ham yetarlicha kichik emas. Bu kuzatishlar sonining kamligi bilan izoxlanadi.

Berilgan korrelasion jadval asosida Y ning X ga regressiya to'g'ri chizig'ining tenglamasi topishda kichik kvadratlar usulida tuzilgan sistema koeffisentlarini ko'paytmalar usulida topishning Maple dasturini tuzamiz.

Kielce: Laboratorium Wiedzy Artur Borcuch

LABORATORIUM WIEDZY
Artur Borcuch

M a p l e   d a s t u r i:

> restart; with(stats):

1)*korrelasion jadval asosida X va Y larini kiritish:*

> X:= Vector([92,96,100,104]); $X := \begin{bmatrix} 92 \\ 96 \\ 100 \\ 104 \end{bmatrix}$

Y:= Vector([158,164,170,176,182]); $Y := \begin{bmatrix} 158 \\ 164 \\ 170 \\ 176 \\ 182 \end{bmatrix}$

2)*korrelasion jadval asosida n_x va n_{xy} chastotalarni kiritish:*

> nx:=Vector([2,7,7,4]); $nx := \begin{bmatrix} 2 \\ 7 \\ 7 \\ 4 \end{bmatrix}$

> nxy:=matrix([[1,1,0,0],[1,4,1,0],[0,2,5,1], [0,0,1,2],[0,0,0,1]]); $nxy := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 2 & 5 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3)*korrelasion jadval asosida shartli o 'rta qiymatlarni hisoblash:*

> Yx[1]:=(Y[1]*nxy[1,1]+Y[2]*nxy[2,1]+ Y[3]*nxy[3,1]+ Y[4]*nxy[4,1]+Y[5]*nxy[5,1])/nx[1];

$Yx_1 := 161$

> Yx[2]:=(Y[1]*nxy[1,2]+Y[2]*nxy[2,2]+
Y[3]*nxy[3,2]+Y[4]*nxy[4,2]+Y[5]*nxy[5,2])/nx[2];

$Yx_2 := \dfrac{1154}{7}$

> evalf(%,4); 164.9

> Yx[3]:=(Y[1]*nxy[1,3]+Y[2]*nxy[2,3]+ Y[3]*nxy[3,3]+ Y[4]*nxy[4,3]+Y[5]*nxy[5,3])/nx[3];

$Yx_3 := 170$

> Yx[4]:=(Y[1]*nxy[1,4]+Y[2]*nxy[2,4]+ Y[3]*nxy[3,4]+ Y[4]*nxy[4,4]+Y[5]*nxy[5,4])/nx[4];

$Yx_4 := 176$

4)*korrelasion jadval asosida X ning qiymatlar soni n va tanlanma xajmi N qiymatlarni kiritish:*

Kielce: Laboratorium Wiedzy Artur Borcuch

> n:=4:N:=20:

5)3- *jadvalning qiymatlarni koʻpaytmalar usulidagi hisoblash:*

> Sx:=add(X[k]*nx[k],k=1..n); $Sx := 1972$

> Sxx:=add(nx[k].X[k]^2,k=1..n); $Sxx := 194704$

> SYx:=add(nx[k].Yx[k],k=1..n); $SYx := 3370$

> SxYx:=add(nx[k].X[k].Yx[k],k=1..n); $SxYx := 332624$

6)*kichik kvadratlar usulida tuzilgan sistemani yechish:*

> ab:=solve([a*Sxx+b*Sx=SxYx,a*Sx+b*N=SYx],{a,b});

$$ab := \left\{ a = \frac{855}{662}, b = \frac{13622}{331} \right\}$$

> evalf(%,4); $\{a = 1.292, b = 41.15\}$

7)*regressiya toʻgʻri chizigʻining tenglamasini yozish:*

> y:=ab[1]*x+ab[2];evalf(%,4);

$$y := x\,a + b = \frac{855}{662}x + \frac{13622}{331} \qquad x\,a + b = 1.292\,x + 41.15$$

2.Berilgan korrelasion jadval asosida *Y* ning *X* ga regressiya toʻgʻri chizigʻining tenglamasi topishda **fit** asfunksiyasidan foydalanib Maple dasturini tuzamiz.

M a p l e d a s t u r i:

> restart;with(stats):

1)1-*korrelasion jadval asosida X va Y larining qiymatlarini chastotalari bilan satr boʻyicha kiritish:*

>W:=[[Weight(92,1),Weight(96,1),Weight(92,1), Weight(96,4),Weight(100,1), Weight(96,2),Weight(100,5), Weight(104,1),Weight(100,1), Weight(104,2), Weight(104,1)],[Weight(158,1), Weight(158,1),Weight(164,1), Weight(164,4), Weight(164,1),Weight(170,2), Weight(170,5), Weight(170,1), Weight(176,1),Weight(176,2), Weight(182,1)]];

$W := [[Weight(92, 1), Weight(96, 1), Weight(92, 1), Weight(96, 4),$
$\quad Weight(100, 1), Weight(96, 2), Weight(100, 5), Weight(104, 1),$
$\quad Weight(100, 1), Weight(104, 2), Weight(104, 1)], [Weight(158,$
$\quad 1), Weight(158, 1), Weight(164, 1), Weight(164, 4), Weight(164,$
$\quad 1), Weight(170, 2), Weight(170, 5), Weight(170, 1), Weight(176,$
$\quad 1), Weight(176, 2), Weight(182, 1)]]$

2) *X va Y larining qiymatlari boʻyicha (x,y) larni koordinatalar sistemasida aniqlash:*

> statplots[scatterplot](W[1],W[2],color=blue, symbol=BOX,symbolsize=20);

(1-rasm)

3)*regressiya toʻgʻri chizigʻining tenglamasini aniqlash:*

> x:=vector(transform[statvalue](W[1]));

Kielce: Laboratorium Wiedzy Artur Borcuch

$x := \begin{bmatrix} 92 & 96 & 92 & 96 & 100 & 96 & 100 & 104 & 100 & 104 & 104 \end{bmatrix}$

> **y:=vector(transform[statvalue](W[2]));**

$y := \begin{bmatrix} 158 & 158 & 164 & 164 & 164 & 170 & 170 & 170 & 176 & 176 & 182 \end{bmatrix}$

> **fit[leastsquare[[x,y]]](W);evalf(%,5);**

$$y = \frac{13622}{331} + \frac{855}{662} x \qquad y = 41.154 + 1.2915 x$$

4)*regressiya to'g'ri chizig'ini qurish:*

> **with(plots):**

> **plot([[x[i],y[i],i=1..11],41.154+1.2915*x], x=90..104, 156..182, style=[point,line],symbol=BOX, color=[red,blue], view=[90..105,150..185], symbolsize=20);** (2-rasm)
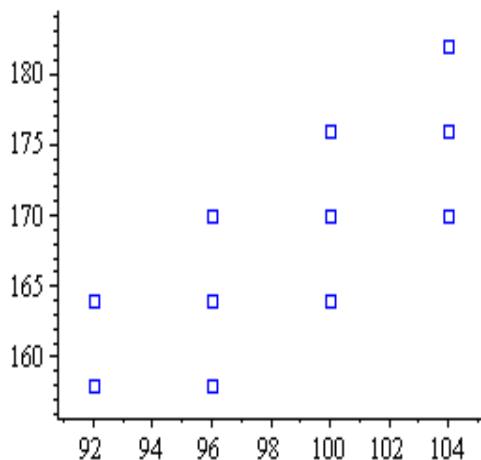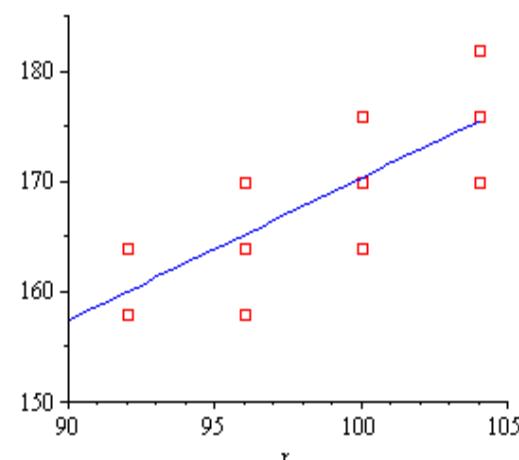
1-rasm.   2-rasm.

XULOSA. Demak, kichik kvadratlar usuli asosida topilgan chiziqli bog'lanish-modeli adikvat bo'lib, uning barcha koeffitsentlari qiymatdor ekanligini topdik. Berilgan tajriba natijalari bo'yicha xulosa va qaror qabul qilish uchun topilgan tajriba natijalari bo'yicha bog'lanish modelni tuzish va samaradorligini aniqlashda ushbu Maple dasturidan foydalanib aniq, tez va sifatli natijalarni olish mumkinligini ko'rdik.

Adabiyotlar

1. Jamshid Faizullayev, Ergashboy Mirzakarimov, Jamshid Mamayusupov, Bobur Tillaboyev and Guljahon Tillaboyeva. (2024) Methods of teaching fundamental sciences based on the integration of information and pedagogical technologies. E3S Web of Conferences (Vol. 538, p. 05012). 14 June 2024.

   https://www.e3s-conferences.org/articles/e3sconf/abs/2024/68/contents/contents. DOI: https://doi.org/10.1051/e3sconf/202453805012.

2. Кремер Н. Ш.,(2001), «Теория вероятностей и математическая статистика». Учебное пособие. Москва.

3. Матросов А.В. (2001). Maple 6. Решение задач высшей математики и механики. СПб.: БХВ-Петербург.

4. Mirzakarimov,E.M.,(2021).Maple tizimida to'g'ri to'rtburchakli membrananing erkin tebranishini aniqlash. *Scientific Bulletin of Namangan State University* 1(8), 3-9.

5. Mirzakarimov,E.M.,(2022).To'g'riburchakli membrananing erkin tebranishini boshlang'ich shartlar bo'yicha aniqlashda maple tizimidan foydalanish. *Scientific Bulletin of Namangan State University* 1(1), 53-61.

6. Mirzakarimov, E.M., & Fayzullaev, J.S. (2020). Improving the quality and efficiency of teaching by developing students* mathematical competence using the animation method of adding vectors to the plane using the maple system. *Scientific Bulletin of Namangan State University*, 2(9), 336-342.

7. Mirzakarimov,E.M., & Faizullaev,J.I. (2019). Method of teaching the integration of information and educational technologies in a heterogeneous parabolic equation. *Scientific Bulletin of Namangan State University*, 1(5), 13-17.

8. Mirzaboevich, M.E., & ugli, E. M.I. (2021). Using Maple Programs in Higher Mathematics Triangle Problem Constructed on Vectors in Space. *Central asian journal of mathematical theory and computer sciences*, 2(11), 44-50. Retrieved from http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/123

9. Мирзабоевич,М.Э., &Исмоилджонович, Ф.Д. (2021). Выполнять Линейные Операции Над Векторами В Пространстве В Системе Maple. *Central asian journal of mathematical theory and computer ciences*, 2(12), 10-16.

   http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS/article/view/137

10. Мирзабоевич,М.Э.,(2022). Использовать Систему Maple Для Определения Свободных Колебаний Прямоугольной Мембраны При Начальных Условиях. *Central Asian Journal Of Mathematical Theory And Computer Sciences*, 3(1),9-18. https://cajmtcs.centralasianstudies.org/index.php/ CAJMTCS/ article/view/148

11. Mirzakarimov E.M. (2018). Oliy matematika fanidan laboratoriya ishlarini Maple dasturida bajarish. T.: Tafakkur bo'stoni.

12. Mirzakarimov , E. ., & Fayzullayev, D. . (2023). Maple tizimida kompleks sonlarlarning geometrik tasviri. *Евразийский журнал математической теории и компьютерных наук*, 3(5), 38–45. https://www.in-academy.uz /index.php/EJMTCS/article/view/14347

13. Mirzakarimov , E. ., & Fayzullayev, D. . (2023). Algebraik shakildagi kompleks sonlar ustida amallarni maple tizimida bajarish**. *Евразийский журнал математической теории и компьютерных наук*, 3(5), 30–37.

    https://in-academy.uz /index.php/EJMTCS/article/view/14346

14. E.M.Mirzakarimov, & J.I.Fayzullayev.(2024). "Geometric representation of complex numbers in the maple system" *Gospodarka i Innowacje*, (45), 70-78,

    https://gospodarkainnowacje-pl.openconference.us/index.php/issue_view_32/ article/view/2373/2195,

15. E.M.Mirzakarimov, & J.I.Fayzullayev.(2024), "Linear operations on complex numbers in the maple system" *Gospodarka i Innowacje.*, (45), 79-86

    https://www.gospodarkainnowacje.pl/index.php/issue_view_32/article/view/2374/2196

16. Mirzakarimov Ergashboy Mirzaboyevich, Fayzullayev Djamshid Ismoiljonovich "Solving Differential Equations Using the Operational Method In the Maple System", Excellencia: International Multi-Disciplinary Journal Of Education, 2024 Volume 02, 212-216, https://multijournals.org/index.php/excel lencia-imje/article/view/514/590.

Kielce: Laboratorium Wiedzy Artur Borcuch

LABORATORIUM WIEDZY
Artur Borcuch