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BIR O'ZGARUVCHILI METAANALITIK TENGLAMA YECHIMINI SOHA CHEGARASINING QISMIDAN DAVOM ETTIRISH MASALASI UCHUN KARLEMAN FORMULASI

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A R T I C L E I N F O.

Kalit so'zlar: kompleks tekislik, metaanalitik, Karleman formulasi, so'ndiruvchi funksiya, silliq yopiq kontur, Koshi tipidagi integral, differentsial operator.

Annotatsiya

Ushbu maqolada metaanalitik funksiya va uning n- tartibli hosilalarini soha chegarasi qismidagi qiymatlariga ko'ra, shu sohaga davom ettirish masalasi qaralgan. Metaanalitik funksiyalar uchun Karleman tipidagi ekvivalent formulalar berilib, ular yordamida metaanalitik funksiyani davom ettirish masalasining yechiluvchanlik kriteriyasi isbotlangan.

D - kompleks tekislikdagi chekli bir bog'lamli, ∂D- silliq oddiy yopiq kontur bilan chegaralangan soha hamda $E \subset \partial D$ yoy bo'lsin.

Agar $w = f(z) = u(x,y) + iv(x,y)$ funksiya D sohada $x\ va\ y$ lar bo'yicha n —tartibgacha uzluksiz hosilalarga ega bo'lib,

$$L_n f(z)=(\partial_{\bar z} - \lambda_n)(\partial_{\bar z} - \lambda_{n-1})\ldots(\partial_{\bar z} - \lambda_1)f(z) = 0 \qquad (1)$$

tenglamani qanoatlantirsa, u holda $f(z)$ funksiya D sohada n —tartibli metaanalitik funksiya deyiladi.

Masalaning qo'yilishi: $f(z)$ funksiya E to'plamda n —metaanalitik funksiyaning berilgan $L_s f(t)$ qiymatlariga ko'ra uni D sohaga davom ettirish talab etiladi. Boshqacha aytganda

$$L_s f(t)= h_s(t)\ , (s=0,1,2,\ldots,n-1\ ) \qquad (2)$$

$$L_0 f(t)=f(t)= h_0(t), t \epsilon E$$

funksiyalarga ko'ra $f(z)$ n —metaanalitik funksiyani D sohaga davom ettirish talab etiladi .

$\lambda_1 = \lambda_2 = \cdots = \lambda_n = 0$ bo'lgan holda (1) va (2) masala n-analitik funksiya uchun chegaraviy davom ettirish masalasiga aylanadi. n-analitik funksiya uchun chegaraviy davom ettirish masalasi [1] ishda o'rganilgan.

(1), (2) masala yechimining yagonaligini isbotlash uchun oldin quyidagi lemmani isbotlaymiz.

Lemma $w(z) \epsilon C(D)$ funksiya D sohada

$$\partial_{\bar z} w - \lambda w = 0 \qquad (3)$$

tenglamani qanoatlantirib, $E \subset \partial D$ yoyda uning chegaraviy qiymatlari nolga aylansa, $w(z) \equiv 0$ bo'ladi.

Isbot. (3) tenglamani integrallaymiz

$$\frac{\partial_{\bar{z}} w}{w} = \lambda \Rightarrow \ln w(z) = \lambda \bar{z} + \ln f(z) \Rightarrow w(z) = f(z) e^{\lambda \bar{z}},$$

Bu yerda $f(z)$ ixtiyoriy golomorf funksiya. Lemma shartiga ko'ra

$f(t) e^{\lambda \bar{t}} = 0, t \epsilon E$ yoki

$f(t) = 0,\ t \epsilon E$.

Golomorf funksiyalar uchun Riss teoremasiga asosan D sohada $f(z) = 0$

ga kelamiz . ya'ni D sohada $w(z)=0$ ekanligi kelib chiqadi. Lemma isbotlandi. D sohoda n-metaanalitik funksiyalar sinfini $M_n(D)$ orqali belgilaymiz.

Endi (1), (2) masala yechimining yagonaligi haqidagi teoremani keltiramiz.

1-Teorema $w(z)\ \epsilon M_n(D) \cap C^{n-1}(\bar{D})$ funksiya $t \epsilon E$ uchun

$L_s w(t) = (\partial_{\bar{t}} - \lambda_s) w(t) = 0,$ s=0,1,2,…,n-1) (4)

shartlarni qanoatlantirsa, u holda D sohada $w(z) \equiv 0$ bo'ladi.

Isbot. $L_{n-1} w(z)$ funksiya (1) ga asosan $(\partial_{\bar{z}} - \lambda_n)\ L_{n-1} w(z)=0,\ z \epsilon D$

tenglamani qanoatlantiradi. (4) ga ko'ra ($s = n - 2$ bo'lganda) uning qiymatlari $E \subset \partial D$ da nolga teng. 1-lemmadan $L_{n-1} w(z)=0,\ z \epsilon D$ (5)

tenglikni olamiz. $L_{n-2} w(z)$ funksiya (5) tenglikga ko'ra

$(\partial_{\bar{z}} - \lambda_{n-1})\ L_{n-2} w(z)=0,\ z \epsilon D$ (6)

tenglamani qanoatlantiradi. (4) ga ko'ra ($s = n - 2$ bo'lganda) uning chegaraviy qiymatlari E da nolga teng. 1-lemmaga asosan

$L_{n-2} w(z)=0,\ z \epsilon D$ tenglamani qanoatlantiradi. Ushbu mulohazani davom ettirib, D sohada $w(z) \equiv 0$ olamiz. Teorema isbotlandi.

Quyidagi misol Adamar misoliga o'xshash bo'lib, (1) va (2) masala yechimi turg'un emasligini ko'rsatadi.

Misol. $w_m(z) = \frac{e^{-imz}}{m} \sum_{k=1}^{n} e^{\lambda_k \bar{z}}$ funksiyalar (1) tenglamani qanoatlantiradi, ya'ni n-metaanalitik bo'ladi. Ularning s-tartibli hosilasi

$$L_s w_m(z) = \sum_{k=s+1}^{n} (\partial_{\bar{z}} - \lambda_s)(\partial_{\bar{z}} - \lambda_{s-1}) \ldots (\partial_{\bar{z}} - \lambda_1) e^{\lambda_k \bar{z}} =$$

$$\frac{e^{-imz}}{m} \sum_{k=s+1}^{n} (\partial_k - \lambda_s)(\partial_k - \lambda_{s-1}) \ldots (\partial_k - \lambda_1) e^{\lambda_k \bar{z}}\ (s = 0,1,2,\ldots,n-1)$$

ga teng bo'lib, $y = 0$ bo'lganda va $m \to \infty\ da$ nolga intiladi.

$$L_s w_m(z)|_{y=0} = \frac{e^{-imx}}{m} \sum_{k=s+1}^{n} (\partial_k - \lambda_s)(\partial_k - \lambda_{s-1}) \ldots (\partial_k - \lambda_1) e^{\lambda_k x} \to 0$$

Lekin yuqori yarim tekislikda ($y > 0$) $w_m(z) \to \infty$, $m \to \infty$ munosabat o'rinli.

Bu misoldan (1), (2) masalaning nokorrekt ekanligi kelib chiqadi.

Analitik funksiyalar nazariyasida Koshi integral formulasi muhum rol o'ynaydi. n-metaanalitik funksiyalar uchun Koshi integral formulasini

V.V. Pokazeyev isbotlagan. [2]

$w(z) \epsilon M_n(D) \cap C^{n-1}(\bar{D})$ funksiya uchun Quyidagi Koshi formulasi o'rinli.

$$f(z) = \frac{1}{2\pi i}\int_{\partial D}\sum_{k=1}^{n} L_{k-1}f(\tau)\sum_{\nu=1}^{k} K_\nu(z-\tau)\prod_{l=1}^{k}\alpha_{\nu l}\frac{d\tau}{\tau-z} \qquad (7)$$

bu yerda $z\epsilon D$, α_{ll}=1, $\alpha_{\nu l}$=$(\lambda_\nu - \lambda_l)^{-1}$ $\nu \neq l$

$K_\nu(z-\tau)$=$e^{[\lambda_\nu\overline{(z-\tau)} - \overline{\lambda_\nu}(z-\tau)]}$.

Koshi formulasidan foydalanib, (1), (2) masalaning yechimini beradigan n-metaanalitik funksiyalar uchun Karleman formulasini hosil qilish mumkin. Shu maqsadda analitik funksiyalar uchun so'ndiruvchi funksiyani qaraymiz.

$$g_\sigma(z)=e^{\sigma h(z)}$$

bu yerda σ-musbat sonli parameter, $h(z) - D$ sohada analitik bo'lib, $Reh(z) = \omega(z)$, $\omega(z) - E$ to'plamning D sohaga nisbatan garmonik o'lchovi.

$w(z)$ $\epsilon M_n(D)$ funksiya uchun $F_\sigma(z) = g_\sigma(z)w(z)$ funksiya ham $M_n(D)$ sinfiga tegishli bo'ladi. Demak, $F_\sigma(z)$ funksiya uchun (7) Koshi formulasi o'rinli.

$$F_\sigma(z) = \frac{1}{2\pi i}\int_{\partial D}\sum_{k=1}^{n} L_{k-1}\, F_\sigma(\tau)\sum_{\nu=1}^{k} K_\nu(z-\tau)\prod_{l=1}^{k}\alpha_{\nu l}\frac{d\tau}{\tau-z} \qquad (8)$$

buni

$$w(z)= \frac{1}{2\pi i}\int_{\partial D}\sum_{k=1}^{n} L_{k-1}w(\tau)\sum_{\nu=1}^{k} K_\nu(z-\tau)\prod_{l=1}^{k}\alpha_{\nu l}e^{\sigma[\alpha(\tau)-\alpha(z)]}\frac{d\tau}{\tau-z}$$

ko'rinishda yozib olib, $z\epsilon D$ nuqtada

$w(z)$- $w_\sigma(z)$=

= $w(z)$- $\frac{1}{2\pi i}\int_{\partial D}\sum_{k=1}^{n} L_{k-1}w(\tau)\sum_{\nu=1}^{k} K_\nu(z-\tau)\prod_{l=1}^{k}\alpha_{\nu l}e^{\sigma[\alpha_\sigma(\tau)-\alpha_\sigma(z)]}\frac{d\tau}{\tau-z}$ ayirmani baholaymiz.

$|w(z) - w_\sigma(z)|$=

$$=\left|\frac{1}{2\pi i}\int_{\partial D\setminus E}\sum_{k=1}^{n} L_{k-1}w(\tau)\sum_{\nu=1}^{k} K_\nu(z-\tau)\prod_{l=1}^{k}\alpha_{\nu l}e^{\sigma[\alpha(\tau)-\alpha(z)]}\frac{d\tau}{\tau-z}\right| \leq$$

$$\leq e^{-\sigma\omega(z)}aK \qquad (9)$$

bu yerda $a = \sum_{\nu=1}^{n}\prod_{l=1}^{k}|\alpha_{\nu l}|$, K= $\max_{\tau\epsilon\,\partial D\setminus E}|L_{k-1}w(\tau)|$, $1\leq k \leq n$.

(9) dan quyidagi teoremaning o'rinli ekanligi kelib chiqadi.

2-Teorema (2) shartlarni qanoatlantiruvchi

$w\, \epsilon M_n(D) \cap C^{n-1}(\bar{D})$, funksiya uchun,

$$w(z)=\lim_{\sigma\to\infty}\frac{1}{2\pi i}\int_E \sum_{k=1}^{n} f_{k-1}(\tau)\sum_{\nu=1}^{k} e^{\left[\lambda_\nu(\bar{z}-\bar{\tau})-\overline{\lambda_\nu}(z-\tau)\right]}\prod_{l=1}^{k}(\lambda_\nu-\lambda_l)^{-1}\ e^{\sigma[\alpha(\tau)-\alpha(z)]}\frac{d\tau}{\tau-z}$$

, $z\epsilon D$ mavjud.

Adabiyotlar:

1. Ишанкулов Т., Фозилов Д. Ш. Продолжение полианалитических функций. // Известия вузов. Математика. **8**, 37-45 (2021). (**Scopus** IF=0.373).
2. Показеев В.И. Интеграл типа Коши для метааналитических функций. // Изв. вузов. Математика. 1982. - № 3. - С. 44-51